

Name _____ Student ID _____

The exam is open book and open notes. You may also use the handouts on “brand name distributions” and Greek letters. You may use a calculator. You may use R on your own computer or Rweb (<https://rweb.webapps.cla.umn.edu/Rweb/Rweb.general.html>)

You **may not** obtain help from any person, computer application, or service other than material on the course web pages or your own notes and homework or R or Rweb. In particular, you are not allowed to use Mathematica or other computer algebra system, including the Wolfram Alpha web site.

Show your work or give an explanation of your answer. No credit for numbers or formulas with no indication of where they came from. Leave no undone derivatives or integrals in your answers, and simplify formulas as much as you easily can, but there is no unique “correct” simplification. Any correct answer gets full credit unless the question explicitly states otherwise.

This exam is on-line. Submit your solutions to the course Canvas site (under Assignments) by 10:10 AM.

Abbreviations used: probability density function (PDF); probability mass function (PMF); distribution function (DF).

The points for the questions total to 100. There are 3 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having PDF given by

$$f(x) = \frac{1}{2} \sin(x), \quad 0 < x < \pi.$$

- (a) Find the mean of X .

- (b) Find the probability of the event $X < 1$.

2. [20 pts.] Suppose X is a random variable having PDF

$$f_{\theta}(x) = \frac{e^{x-\theta}}{(1 + e^{x-\theta})^2}, \quad -\infty < x < \infty.$$

Find the PDF of the random variable

$$Y = \frac{1}{1 + e^{-X}}.$$

The definition of a function describes the domain as well as the rule.

3. [20 pts.] Suppose X is a random variable having DF given by

$$F_{\theta}(x) = \frac{1}{1 + e^{\theta-x}}, \quad -\infty < x < \infty,$$

where $-\infty < \theta < \infty$.

(a) Find its PDF.

(b) Show that the distribution of X is symmetric about θ .

4. [20 pts.] Suppose the random vector (X, Y) has the PDF

$$f_{\alpha, \beta}(x, y) = \frac{6\alpha\beta}{(1 + \alpha x + \beta y)^4} \quad 0 < x < \infty, \quad 0 < y < \infty,$$

where $\alpha > 0$ and $\beta > 0$ are parameters.

- (a) Find the marginal PDF of X . (Hint: do the integral by substitution.)

- (b) Find the conditional PDF of Y given X .

- (c) Find the conditional expectation of Y given X . (Hint: do the integral by substitution.)

5. [20 pts.] Suppose X is continuous, Y is discrete, and the conditional PMF of Y given X is

$$f(y | x) = \frac{\Gamma(\theta + y)}{\Gamma(\theta)y!} x^\theta (1 - x)^y, \quad y = 0, 1, 2, \dots$$

where $\theta > 0$ and $0 < x < 1$, and suppose the marginal distribution of X is Beta(α_1, α_2) where $\alpha_1 > 0$ and $\alpha_2 > 0$. What is the conditional distribution of X given Y ?