Stat 5101 First Midterm Exam **October 14, 2020**

Name ______________________ Student ID ______________

The exam is open book and open notes. You may also use the handouts on "brand name distributions" and Greek letters. You may use a calculator.

You **may not** obtain help from any person, computer application, or service other than material on the course web pages or your own notes and homework. In particular, you are not allowed to use Mathematica or other computer algebra system, including the Wolfram Alpha web site.

Show your work or give an explanation of your answer. No credit for numbers or formulas with no indication of where they came from. Simplify formulas as much as you easily can, but there is no unique "correct" simplification. Any correct answer gets full credit unless the question explicitly states otherwise.

This exam is on-line. Submit your solutions to the course Canvas site (under Assignments) by 10:10 AM.

Abbreviations used: probability mass function (PMF).

The points for the questions total to 100. There are 3 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having PMF with parameter $\theta > 0$ given by

$$f(x) = \frac{\theta^x}{\theta^3 + \theta^4}, \qquad x = 3, 4.$$

In this problem simplify your answers so they do not leave undone a sum over the points in the sample space.

Hint: Consider $Y = X - 3$.

(a) Calculate $E_\theta(X)$.

(b) Calculate $\operatorname{var}_\theta(X)$.

2. [20 pts.] Suppose X is a random variable having the uniform distribution on the set $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. Find the PMF of the random variable $Y = X^2 - 4$.

3. [20 pts.] Suppose $\mathbf{X}$ is a random vector with mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & 0 & 0 & 0 \\ m_{12} & m_{11} & m_{12} & 0 & 0 \\ 0 & m_{12} & m_{11} & m_{12} & 0 \\ 0 & 0 & m_{12} & m_{11} & m_{12} \\ 0 & 0 & 0 & m_{12} & m_{11} \end{pmatrix}$$

where μ_i, $i = 1, \ldots, 5$ and m_{11} and m_{12} are real numbers satisfying

$$\begin{aligned} m_{11} &\geq 0 \\ m_{11}^2 &\geq 3m_{12}^2 \end{aligned}$$

(a) Find $E(X_1 + X_2 + X_3 + X_4)$.

(b) Find $\operatorname{var}(X_1 + X_2 + X_3 + X_4)$.

4. [20 pts.] Define $S = \{0, 1, 2, 3, 4\}$. Each of the following functions is the PMF of a random vector. For each say whether the components of that random vector are independent or dependent, and say why.

   (a) $f(x, y) = \frac{256}{961} \cdot 2^{-x-y}, \qquad (x, y) \in S^2.$

   (b) $f(x, y) = xy/100, \qquad (x, y) \in S^2.$

   (c) $f(x, y) = (x + y)/100, \qquad (x, y) \in S^2.$

5. [20 pts.] In this problem, simplify your answers so they do not contain any unevaluated binomial coefficients.

   (a) There are 4 blue and 2 white balls in an urn, and we draw a random sample of size 3 with replacement from the urn (this means the balls are well mixed before each draw). If X is the number of white balls drawn, what is the PMF of X?

   (b) Exactly the same question as in part (a) except that we change with replacement to without replacement (in which case it does not matter whether the balls are well mixed between draws as long as they were well mixed before the first draw).