

Stat 5101 Second Midterm Exam

April 6, 2022

Name _____ Student ID _____

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on “brand name distributions” and Greek letters. You may use a calculator. No other electronic devices are allowed.

Put all of your work on this test form (use the backs of pages if necessary). Show your work or give an explanation of your answer. No credit for numbers or formulas with no indication of where they came from. Leave no undone sums or integrals for probabilities or expectations in your answers. But other than that requirement, there is no unique “correct” simplification for any answer. Any correct (and explained) answer gets full credit unless the question explicitly states otherwise.

Abbreviations used: probability density function (PDF); probability mass function (PMF); distribution function (DF).

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having PDF given by

$$f(x) = \frac{\alpha + \beta x}{\alpha + \beta/2}, \quad 0 < x < 1,$$

where α and β are positive parameters.

(a) Find $E(X)$.

(b) Find $E(X^2)$.

(c) Find $\text{var}(X)$.

2. [20 pts.] Suppose X is a random variable having PDF given by

$$f(x) = \frac{\alpha + \beta x^2}{\alpha + \beta/3}, \quad 0 < x < 1,$$

where α and β are positive parameters. Find the PDF of the random variable

$$Y = \frac{1}{X}$$

The definition of a function describes the domain as well as the rule.

3. [20 pts.] Suppose X is a random variable having PDF given by

$$f(x) = \frac{\alpha + \beta x^3}{\alpha + \beta/4}, \quad 0 < x < 1,$$

where α and β are positive parameters.

Find the DF of X . Be sure to define the DF on the whole real line.

4. [20 pts.] Suppose the random vector (X, Y) has the PDF

$$f(x, y) = \frac{\alpha + \beta x + \gamma y + \delta xy^2}{\alpha + \beta/2 + \gamma/2 + \delta/6} \quad 0 < x < 1, 0 < y < 1,$$

where α , β , γ , and δ are positive parameters.

(a) Find the marginal PDF of X .

(b) Find the conditional PDF of Y given X .

(c) Find the conditional expectation of Y given X .

5. [20 pts.] Suppose the conditional distribution of Y given X is $\mathcal{N}(0, 1/X)$ and the marginal distribution of X is $\text{Exp}(\lambda)$. (Note that the second parameter of the normal distribution, here $1/X$, is the *variance* not the standard deviation.) What is the conditional distribution of X given Y ?