4-1. Show the following

(a) $O_p(X_n)O_p(Y_n) = O_p(X_nY_n)$.
(b) $O_p(X_n)o_p(Y_n) = o_p(X_nY_n)$.
(c) $o_p(X_n)o_p(Y_n) = o_p(X_nY_n)$.
(d) If $X_n = o_p(1)$, then $o(O_p(X_n)) = o_p(X_n)$.

You may assume that all random elements involved in this problem are real-valued (multiplication of vectors is undefined). These problems are to show that the left-hand side can be replaced by the right-hand side, that is, for (a), if $U_n = O_p(X_n)$ and $V_n = O_p(Y_n)$, then $U_nV_n = O_p(X_nY_n)$.

4-2. A problem unrelated to anything (came up in research, just want to see what everyone does). Suppose $X$ is a random element of $\mathbb{R}^d$ having PDF $f_\theta$ and define $Y = I_A(X)$, where $A$ is some measurable subset of $\mathbb{R}^d$. This problem is about the joint distribution of the pair $(X,Y)$ and its marginals and conditionals. Since $X$ is continuous and $Y$ is discrete, this is a bit tricky. In measure-theoretic terms you can give a density with respect to the product of Lebesgue measure on $\mathbb{R}^d$ and counting measure on $\{0,1\}$. If you haven’t had measure theory, you can still do this just intuitively treating $X$ as continuous and $Y$ as discrete. If $g_\theta$ is the joint density of $X$ and $Y$ (that we are trying to find) then the expectation of any function $h$ is given by

$$\sum_{y=0}^1 \int h(x,y)g_\theta(x,y)\,dx$$

(integrate over the continuous variable and sum over the discrete variable).

(a) Give the joint density $g_\theta$ of the pair $(X,Y)$.
(b) Give the marginal density of $X$ and the conditional density of $Y$ given $X$.
(c) Give the marginal density of $Y$ and the conditional density of $X$ given $Y$.

4-3. Ferguson, Problem 9-2.

4-4. Ferguson, Problem 10-1.
4-5. Ferguson, Problem 10-2.
4-6. Ferguson, Problem 13-1.
4-7. Ferguson, Problem 13-2.
4-8. Ferguson, Problem 13-4.