1. [20 pts.] Suppose $X$ is a random variable having PMF with parameter $\theta \geq 0$ given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_\theta(x)$</td>
<td>$\frac{\theta^2}{1 + 2\theta + 2\theta^2}$</td>
<td>$\frac{\theta}{1 + 2\theta + 2\theta^2}$</td>
<td>$\frac{1}{1 + 2\theta + 2\theta^2}$</td>
<td>$\frac{\theta}{1 + 2\theta + 2\theta^2}$</td>
<td>$\frac{\theta^2}{1 + 2\theta + 2\theta^2}$</td>
</tr>
</tbody>
</table>

In this problem simplify your answers so they do not leave undone a sum over the points in the sample space.

(a) Calculate $E_\theta(X)$.

(b) Calculate $\text{var}_\theta(X)$. 
2. [20 pts.] Suppose $X$ is a random variable having PMF with parameter $\theta \geq 0$ given by

\[
\begin{array}{c|ccccc}
  x & 0 & 1 & 2 & 3 & 4 \\
  f_\theta(x) & \frac{1-\theta}{1-\theta^5} & \frac{(1-\theta)\theta}{1-\theta^5} & \frac{(1-\theta)\theta^2}{1-\theta^5} & \frac{(1-\theta)\theta^3}{1-\theta^5} & \frac{(1-\theta)\theta^4}{1-\theta^5} \\
\end{array}
\]

Find the PMF of the random variable $Y = X(X - 2)(X - 4)$. 

3. [20 pts.] Suppose $X$ is a random vector with mean vector

$$
\mu = \begin{pmatrix}
\mu_1 \\
\mu_1 \\
\mu_2 \\
\mu_2
\end{pmatrix}
$$

and variance matrix

$$
M = \begin{pmatrix}
\sigma_1^2 & \rho_1 & 0 & 0 \\
\rho_1 & \sigma_1^2 & 0 & 0 \\
0 & 0 & \sigma_2^2 & \rho_2 \\
0 & 0 & \rho_2 & \sigma_2^2
\end{pmatrix}
$$

where $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho_1, \rho_2$ are real numbers satisfying $|\rho_1| \leq \sigma_1^2$ and $|\rho_2| \leq \sigma_2^2$.

(a) Find $E(X_1 + X_2 + X_3 + X_4)$.

(b) Find $\text{var}(X_1 + X_2 + X_3 + X_4)$.
4. [20 pts.] Suppose $X$ and $Y$ are independent and uniformly distributed on the set $S = \{1, 2, 3\}$. In each part give reasoning justifying your answer.

(a) Are $X^2$ and $Y^3$ independent?

(b) Are $X$ and $X + Y$ independent?

(c) Are $X - Y$ and $X + Y$ independent?
5. [20 pts.] In this problem, simplify your answers so they do not contain any unevaluated binomial coefficients.

(a) There are 3 red and 1 white balls in an urn, and we draw a random sample of size 2 with replacement from the urn (this means the balls are well mixed before each draw). If $X$ is the number of red balls drawn, what is the PMF of $X$?

(b) Exactly the same question as in part (a) except that we change with replacement to without replacement (in which case it does not matter whether the balls are well mixed between draws as long as they were well mixed before the first draw).