Solutions for Assignment 9

Stat 3011, Summer 2006

Seongho Wu

1 Problem 14.2

(a) If $\mu = 115$, the distribution is approximately Normal with mean $\mu = 115$ and standard deviation $\sigma/\sqrt{25} = 6$.
(b) The actual result lies out toward the high tail of the curve, while 118.6 is fairly close to the middle. If $\mu = 115$, observing a value similar to 118.6 would not be too surprising, but 125.8 is less likely, and it therefore provides some evidence that $\mu > 115$.

2 Problem 14.4

$$H_0 : \mu = 115 \quad vs. \quad H_a : \mu > 115.$$

3 Problem 14.9

For $\bar{x} = 118.6, z = \frac{118.6-115}{30/\sqrt{25}} = 0.60$. For $\bar{x} = 125.8, z = \frac{125.8-115}{30/\sqrt{25}} = 1.80$.

4 Problem 14.12

For $\bar{x} = 118.6$, the P-value is $P(\bar{x} > 118.6) = P(Z > 0.6) = 0.2743$. For $\bar{x} = 125.8$, P $= P(\bar{x} > 125.8) = P(Z > 1.80) = 0.0359$. A small P-value (such as 0.0359) tells us that values of $\bar{x}$ similar to 125.8 would rarely occur when H_0 is true, while P $= 0.2743$ indicates the results similar to 118.6 give little reason to doubt H_0.

5 Problem 14.15

For $\bar{x} = 125.8$, we found $z = 1.80$ and P $= 0.0359$; this is significant at $\alpha = 0.05$, but not at $\alpha = 0.01$.

6 Problem 14.26

For testing $H_0 : \mu = 150\,\text{min} \quad vs. \quad H_a : \mu < 150\,\text{min}$, the test statistic is

$$z = \frac{137 - 150}{65/\sqrt{269}} = -3.28,$$

and the P-value is P $= P(Z < -3.28) = 0.0005$. This is very strong evidence that students study less than 2.5 hours per night.

7 Problem 14.27

Our hypotheses are $H_0 : \mu = 100 \quad vs. \quad H_a : \mu \neq 100$. We find that $\bar{x} = 105.84$, so the test statistic is $z = \frac{105.84-100}{15/\sqrt{31}} = 2.17$, and the P-value is P $= 2P(Z > 2.17) = 0.0300$. This is strong evidence (significant at the 5% level) that the mean IQ differs from (is greater than) 100.

8 Problem 14.28

We test $H_0 : \mu = 25 \quad vs. \quad H_a : \mu > 25$. We find that $\bar{x} = 30.4$, and the test statistic is $z = \frac{30.4-25}{7/\sqrt{10}} = 2.44$, so the P-value is P $= P(Z > 2.44) = 0.0073$. This is strong evidence against H_0; we conclude that the student's mean threshold is greater than 25μg/l.

9 Problem 14.37

(a) Yes: In order to reject H_0 at $\alpha = 0.05$, we must see results that would occur less than 5% of the time if H_0 were true. In fact, our results would occur only 3.3% of the time if H_0 were true.
(b) No: A level $\alpha = 0.01$ test requires stronger evidence than 5% test; specifically, we must observe results that would occur only 1% of the time when H_0 is true.

10 Problem 14.39

P = 0.02 means that if there were truly no change in cholesterol-if all differences were due to random fluctuation or "change assignment"-then only 2% of all samples would produce results similar to the ones found in this sample.

11 Problem 14.48

We find the interval to be 29,737 to 31,945 pounds.
(a) Because 32,000 pounds is not in this interval, we would reject $H_0 : \mu = 32,000$ pounds at the 10% level (in favor of $H_a : \mu \neq 32,000$ pounds).
(b) Because 31,500 pounds is in this interval, we cannot reject $H_0 : \mu = 31,500$ pounds at the 10% level.

12 Problem 15.6

(a) With $n = 100, z = \frac{478-475}{100/\sqrt{100}} = 0.3$, so P $= P(Z > 0.3) = 0.3821$.
(b) With $n = 1000, z = \frac{478-475}{100/\sqrt{1000}} = 0.95$, so P $= P(Z > 0.95) = 0.1711$.
(c) With $n = 10,000, z = \frac{478-475}{100/\sqrt{10000}} = 3$, so P $= P(Z > 3) = 0.0013$.

13 problem 15.7

The interval is $478 \pm (2.576)(100/\sqrt{n})$. The results are as follows:

$$n = 100 \quad (452.24, 503.76)$$
$$n = 1000 \quad (469.85, 486.15)$$
$$n = 10,000 \quad (454.72, 480.58)$$

14 Problem 15.13

(a) H_0: The patient is ill (or "the patient should see a doctor"); H_a: The patient is healthy (or "the patient should not see a doctor"). A Type I error means a false negative-clearing a patient who should be referred to a doctor. A Type II error is a false positive-sending a healthy patient to the doctor.
(b) One might wish to lower the probability of a false negative so that most ill patients are treated. On the other hand, if money is an issue, or there is concern about sending too many patients to see the doctor, lowering the probability of false positives might be desirable.