Final Examination, Stat 1001, Solution

This is a closed book exam. However, you are permitted to have a four-page summary of notes (front and back OK). Each problem or each part of a problem is worth 10 points (maximum score is 180). Calculators are permitted. Put your answers directly on this exam.

1. In a study to compare two treatment conditions, what is the difference between an *experiment* and an *observational study*?

   An experiment includes random assignment of treatments to subject; an observational study uses assignment beyond the control of the experimenter.

2. In a recent court case in the United Kingdom, a woman was tried for murdering her two young children. The defense claimed that murder was not committed, but rather that the children were victims of sudden infant death syndrome, or SIDS, a condition that leaves children dead for no apparent reason. The chance that one child dies due to SIDS in the UK is known to be about 1/8000.

   Using this value, an expert for the prosecution testified that the chance that two children in the same family die due to SIDS is

   $$\frac{1}{8000} \times \frac{1}{8000} = \frac{1}{64{,}000{,}000}$$

   Since this probability is so small, the expert suggested that SIDS could not be the cause of the children's deaths.

   (a) What rule of probability did the expert use to make this calculation?
   Independence of events.

   (b) If you were working with the defense, what arguments would you use to discredit the expert's testimony?
   There is no basis presented for assuming independence, and this probability could be much larger. The chance that two children die of SIDS in some family, as opposed to a specific family, is of course much larger still, as in the birthday problem.

3. Which of the following does a hypothesis test deal with (check all those that apply):

   _____ Is the difference due to chance?
   _____ Is the difference important?
   _____ What does the difference prove?
   _____ Was the experiment properly designed?

4. You do a survey of $n = 100$ suburban high school graduates who competed high school between 1990 and 1995, and then enrolled at the University of Minnesota. Of these, 65 graduated from the University is 6 years or less.

   (a) Give a 95% confidence interval for the population fraction of 1990-95 suburban high school graduates who enrolled at the University of Minnesota and graduated in six years or less.
   The point estimate is $\hat{p} = 65/100 = .65$ *and the standard error of the estimate is* $\sqrt{.65 \times .35/100} \approx .05$*, so a 95% confidence interval for* p *is from* $.65 - 2 \times .05$ *to* $.65 + 2 \times .05$*, or from* $.55$ *to* $.75$.

   (b) Write a sentence that explains the meaning of a 95% confidence interval for the probability of success p (you can use the results from Question 4a if you wish).
   If we compute confidence intervals using the methodology used in Question 4a over and over, 95% of the intervals will correctly include the true value.

5. Of those who entered the University of Minnesota as Freshmen in 1994, 80% also enrolled in 1995. We would say that the *retention rate* is 80%. If you choose two students at random from among those who enrolled as freshman in 1994, what is the probability that at least one of them returned in 1995?

   The chance of at least one success is one minus the probability of two failures, or $1 - .2 \times .2 = 0.96$.

6. Our lab has two machines A and B that measure the moisture content of samples of wood. We want to know if machines produce similar measurements, so we select 50 samples of wood at random from a population and measure the moisture content of each sample on each machine. We then compute the sample correlation between the measurement on A and the measurement on B for each sample.

   (a) What value of the correlation would you expect if the machines are measuring the same thing?

   Correlation close to $+1$.

   (b) Suppose the sample correlation turned out to be close to zero. How would you explain this result?

   This suggests that the machines are not measuring the same thing.

7. A major court case on the health effects of drinking contaminated water took place in the town of Woburn, Massachusetts. A town well in Woburn was contaminated by industrial chemicals. During the period that residents drank water from this well, there were 16 birth defects among 414 births. In the years when the contaminated well was shut off and water was supplied from elsewhere, there were 3 birth defects among 228 births.

   (a) What is the relative risk of birth defects when drinking contaminated well water when compared to drinking clean water?

   $(16/414)/(3/228) = 2.93$.

   (b) For these data, the value of $X^2 = 3.31$. Do the data support the claim that drinking contaminated water is associated with an increase in birth defects? Why or why not?

   Since $X^2 < 3.84$, *the* p*-value for the hypothesis test of no effect versus an effect would not be rejected at the 5% level.*

8. The Donner Party was one of the most famous tragedies in the history of the westward migration in the United States. In the winter of 1846-47, about 91 emigrants in covered wagons were unable to cross the Sierra Nevada Mountains of California before winter. In the end, 42 people starved to death, and only 49 survived. Figure 1 is a boxplot that summarizes information about the ages of the survivors and those who died in the Donner party.

   (a) Describe the important characteristics of the distribution of ages of the survivors.

   The median survival age was about 16 years, with half the survivors between about 12 and 20. Only one survivor exceeded age about 42.

   (b) Summarize a comparison of the age distribution of the survivors and those who died.

   Those who died had a higher median age, about 22 years, but their ages were more variable. It seems that both the very young and the old died, and the survivors were more nearly constant in age.

9. Saccharin is used as an artificial low-calorie sweetener in diet soft drinks. There is some concern that it may cause cancer. Investigators did a study using rats, in which rats were allocated at random to a treatment group getting 2% of their daily food intake in the form

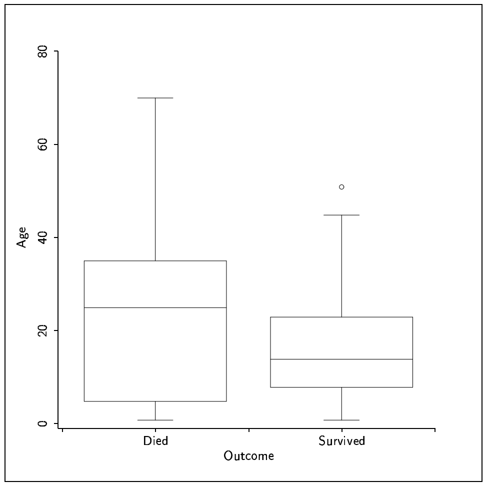

Figure 1: Figure for Problem 8.

of saccharin, or a control group that got no saccharin. The response was the fraction of each group that get bladder cancer. The observed fraction of cancers is higher in the saccharin group than in the control group.

(a) What are the null and alternative hypothesis?

Null hypothesis: Rates are the same for the two groups. Alternative hypothesis: The rates are different.

(b) Based on the data, suppose that the p-value is .0001. Write a sentence that summarizes the information in this number in terms of the problem.

This provides strong evidence against the null hypothesis of no difference, suggesting that the cancer rates are different in the two groups.

10. You have just opened a factory (congratulations) in which you plan to manufacture high-precision widgets. You hope to make the best and most consistent widget in the industry.

Widgets are used to make a device called a *framus*. There are 9 widgets in a framus, and a finished framus will be unacceptable if the average weight of the nine widgets is either less than 9 grams or more than 11 grams, since then the framus will fail shortly after it is installed. You know from prior experience that widget weight approximately follows a bell-shaped curve, with mean 10 grams and SD 2 grams. Compute the fraction of framuses that will be unacceptable.

The standard error of the mean of 9 observations is $2/\sqrt{9} = .67$. *We want the area under a normal curve to the left of* $z_1 = (9 - 10)/.67 = -1.5$ *and the area to the right of* $z_2 = (11 - 10)/.67 = +1.5$. *Each of these is about 0.07. So, about .14 or 14% of the framuses will be unacceptable. You should try to change your manufacturing process to make the standard deviation smaller, or you are likely to go bankrupt.*

11. A new teaching method is compared to a standard method in 50 classrooms with 25 classrooms allocated to each method. Here were the results:

Group	n	Average	SD
Treatment	25	78	5
Control	25	60	6

(a) Give a 95% confidence interval for the population difference in the means of the two groups.

The standard error of a difference is $\sqrt{5^2/25 + 6^2/25} = \sqrt{2.2} \approx 1.5$, *so the confidence interval is from* $18 - 2 \times 1.5$ *to* $18 + 2 \times 1.5$, *or* 15 *to* 21.

(b) True or false: If we were to choose two new classrooms at random, and give one the treatment and one control, then the chance that the difference between the two classrooms is in the confidence interval is 95%.

False.