THE UNIVERSITY OF MINNESOTA

Statistics 5401 September 26, 2005

Assignment Sheet No. 3

Reading

Week of September 26 - September 30: J&W Chapter 5, 6.1-6.2
Week of October 3 - 7: J&W Sec. 6.3 - 6.9

Written Assignment (due in class Monday, October 3):

Here is a listing (edited here to save space) of file `scores.txt`. There is a link to `scores.txt` on the web page

`http://www.stat.umn.edu/~kb/classes/5401/datafiles.html`.

Click with the right button (Windows) or click and hold (Macintosh) to bring up a menu. Choose **Save As** or **Save Target** to save it to your disk.

```
readmath      20     5 format
) Data from experiment comparing standard educational program
) (group 1) with new "reading & math" program (group 2)
) 10 children in each group.
) Col. 1: grp = group no. (1 or 2)
) Col. 2: r1 = pre-program reading score
) Col. 3: r2 = post-program reading score
) Col. 4: m1 = pre-program math score
) Col. 5: m2 = post-program math score
)"%lf %lf %lf %lf %lf"
(f2.0,4f4.1)  In the file itself there are only 5 columns
 1 3.2 4.1 4.7 5.5 |  2 5.0 6.1 5.1 7.1
 1 4.2 4.6 4.5 5.0 |  2 5.2 6.3 5.2 7.0
 1 4.5 4.8 4.6 6.0 |  2 5.3 6.5 5.6 6.2
 1 4.6 5.4 4.8 6.2 |  2 5.4 6.7 5.7 6.8
 1 4.9 5.2 4.9 6.1 |  2 5.8 7.0 5.9 7.1
 1 4.8 5.7 5.0 5.9 |  2 4.8 6.5 5.1 6.9
 1 4.9 6.0 5.1 6.0 |  2 5.9 7.1 6.1 6.7
 1 5.0 5.9 6.0 6.1 |  2 5.0 6.9 4.8 7.0
 1 4.2 4.6 4.5 5.0 |  2 5.6 6.7 5.1 6.9
 1 3.3 4.2 4.8 5.2 |  2 5.7 7.2 6.0 7.4
```

You can read the data in MacAnova by

```
Cmd> scores <- read("","readmath") # read scores.txt
```

The data were obtained in an experiment intended to compare a new "reading and mathematics" curriculum with a standard (control) curriculum. Out of 20 children participating in the study, 10 were randomly assigned for a semester to a class with the new curriculum and the remaining 10 to the standard curriculum. Thus the experimenter used a completely randomized design.

Each child was given a standardized test for Reading (R) and and a standardized test for Math (M) at the start of the semester (pre-test) and again at the end of the semester, after the program (post-test). Tests R and M were intended to measure reading proficiency and mathematics skills, respectively.

Thus there were four variables (test scores) measured for each child:

R_1 = pre-program reading score M_1 = pre-program math score
R_2 = post-program reading score M_2 = post-program math score

This is an example of a *repeated measures design*. As always, you should routinely check for non-normality. If you find evidence, comment on it but attempt *no* remedial action.

1. (a) For the control group, use Hotelling's T^2 to test the null hypothesis that the population mean R and M scores did not change during the semester, that is, that $E[R_2 - R_1]$ and $E[M_2 - M_1]$ are both zero in this group. State the null hypothesis symbolically in terms of the population mean vector for the control group. Use $\alpha = .05$.

Note: Probably the most straightforward way to compute T^2 is to construct the 10 by 2 matrix consisting of the post minus pre differences and then compute a one sample T^2 statistic to test the hypothesis that the means are zero. Alternatively, if you compute a 4 by 1 column vector $\bar{\mathbf{x}}$ and a 4 by 4 unbiased covariance matrix $\mathbf{S}$ from the 10 by 4 group 1 data, you can compute the T^2 for H_0: $\mathbf{L}'\boldsymbol{\mu} = 0$, where $\mathbf{L} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$ as

$T^2 = n(\mathbf{L}\bar{\mathbf{x}})'(\mathbf{L}'\mathbf{S}\mathbf{L})^{-1}(\mathbf{L}\bar{\mathbf{x}})$ using the fact that $\hat{V}[\mathbf{L}'\mathbf{x}] = \mathbf{L}'\mathbf{S}\mathbf{L}$ and hence $\hat{V}[\mathbf{L}'\bar{\mathbf{x}}] = n^{-1}\mathbf{L}'\mathbf{S}\mathbf{L}$. This is not very different from the first method, since $\mathbf{y} = \mathbf{L}'\mathbf{x}$ is simply the 2 by 1 vector of post minus pre differences.

1. (b) Plot a 95% confidence ellipse for the expected change (post – pre) in the scores for the control group. You can do this by hand or by using one of the representations such as the following, for the boundary of the ellipse.

<u>Note</u>: When p = 2, the boundary of the ellipse (in the w_1 - w_2 plane) defined by

$$\{\mathbf{w} \mid (\mathbf{w} - \mathbf{w}_0)'\mathbf{A}^{-1}(\mathbf{w} - \mathbf{w}_0) = c^2\},\ \mathbf{w}_0 = [w_{10}, w_{20}]',\ \mathbf{A} = [a_{ij}], \text{ a 2 by 2 matrix}$$

consists of points $\mathbf{w} = [w_1, w_2]$, where w_2 satisfies

$$w_2 = w_{2\,0} + \{a_{12}(w_1 - w_{10}) \pm c\{(a_{11}a_{22} - a_{12}^2)(a_{11} - (w_1 - w_{10})^2/c^2)\}^{1/2}\}/a_{11}.$$

What are $\mathbf{A}$, $\mathbf{w}_0$, and c^2 for this problem?

See also the handouts on *The Shapes of Ellipses and Ellipsoids* and *Drawing Ellipses in MacAnova*. Macro `ellipse()` distributed with MacAnova provides an easier way based on one of the methods discussed in the handout. Type

```
help(ellipse:vector("usage","plotting"))
```

to get a summary of how it is used.

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Remark: Notation such as "{**w** | *conditions on w*}" is short hand for "the set of all **w** satisfying the *conditions on w*." Thus $\{\mathbf{x} \mid \|\mathbf{x}\|^2 = 1, x_1 \geq 0\}$ means the set of all **x**'s with $\|\mathbf{x}\|^2 = 1$ and $x_1 \geq 0$.

1. (continued)
(c) Find 95% simultaneous confidence intervals for the R and M post-pre means in two ways, using the Bonferroni inequality and intervals based on the ellipsoid.

2. (a) Use a two-sample Hotelling's T^2 to test the hypothesis that the effect of the new curriculum was not different from the effect of the standard curriculum. Should this be based only on the post-program scores (R_2 and M_2) or on a comparison between the two groups of the mean {post – pre} scores ($R_2 - R_2$ and $M_2 - M_1$)? Justify your choice.

2. (b) Test the same hypothesis as in (a) using Bonferronized univariate t tests. What specific advantage is there in examining these t-statistics that T^2 does not possess?

3. Do problem 5.20 on pp. 268 of J&W. The data are in matrix `T05_12` in file `JWData5.txt`.