Displays for Statistics 5401

Lecture 1

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Class Web Page
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Multivariate vs Univariate Statistics

Traditional statistics deals with only *one* variable such as **height**, **survival time** or **crop yield**, at a time.

Such an approach is *univariate*.

Exception? **Multiple regression analysis** where you predict y on the basis of k variables x_1, ..., x_k using a model like

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + e$$

No! This is part of univariate statistics because there is only *one* **response** variable even though there are *many* **predictor** variables.

Stat 5421 includes some **multivariate categorical data analysis**. Before summarizing in a contingency table, data consist of "levels" of p different categories for each of many cases. This is truly multivariate. We won't explore multivariate categorical data at all.

Multivariate statistics emphasizes the **simultaneous** analysis of **more than one** response variables

$$X_1, X_2, \ldots, X_p, \text{ with } p > 1$$

measured or observed on a *single experimental or observational "unit"* such as a person, a tree, a plot or a classroom.

Multivariate statistics often makes inference about a whole vector $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_k]'$ of parameters at once, that is, simultaneously. For example several methods result in inference about a mean vector

$$\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_p]'$$

Univariate statistics would make separate inferences about each μ_j.

We generally use the notation

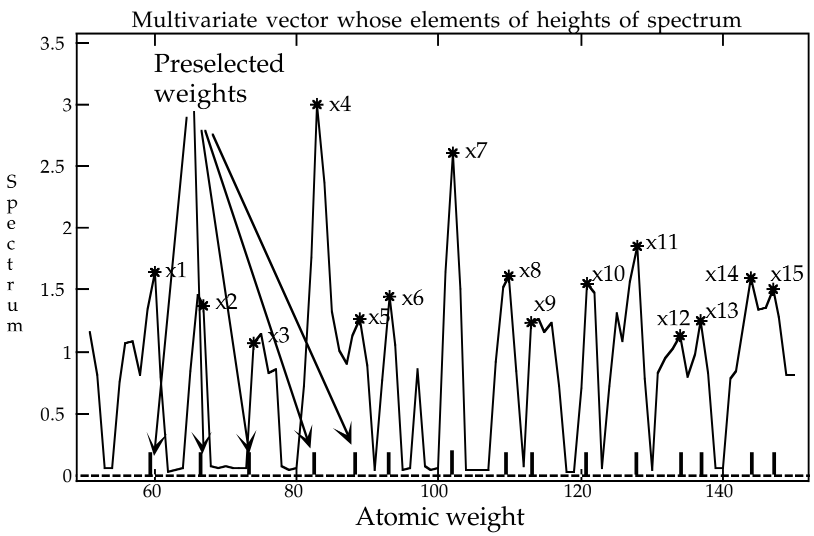
$p \equiv$ number of variables
n or $N \equiv$ number of cases

Examples:

- $p = 3$ measurements on a tree:
  X_1 = DBH = diameter at breast height
  X_2 = height of tree
  X_3 = age of tree
- $p = 5$ anthropometric measurements
  X_1 = body weight X_4 = femur length
  X_2 = body height X_5 = tibia length
  X_3 = skull height
- $p = 4$ scores on "battery" of tests taken by an individual
  X_1 = score on math aptitude test
  X_2 = score on abstract thinking test
  X_3 = score on verbal aptitude test
  X_4 = score on anxiety profile test
- $p = 75$ ratings on each of 75 items on a questionnaire.

- Heights of lines in a "spectrum": $X_1,\ldots,X_p$ = measurements of intensity (height) of spectrum at p specific frequencies or molecular weights of interest.

Example with p = 15. The location of the peaks was *chosen in advance*.



These are not real data.

- **Repeated measures**
  p = 6 measurements of heart rate on the *same* individual every four hours for a day at 0400h, 0800h, 1200h, 1600h 2000h, 2400h.

All these examples represent multiple data items for the *same* individual or experimental/observational unit.

Note For measurements x_1, x_2, ..., x_p to be considered **repeated measures**, all x_i must be *directly comparable*.

This means they are determinations of the *same* quantity at *different* times or under different conditions.

- A person's height and weight *doesn't* constitute repeated measures data.
- A person's height at ages 1, 2, 3, 5, 10 and 15 *would* be repeated measures data.
- A persons heart rate after jogging 1/4 mile, 1/2 mile, 1 mile, & 2 miles.

Sometimes one or more *subsets* of variables *are* repeated measures, but the *whole set* of variables is not.

Example (p = 14):

- $x_1 \ldots x_6$ = systolic blood pressure every 4 hours (one subset of repeated measures variables)
- $x_7 \ldots x_{12}$ = heart rate every 4 hours (second subset of repeated measures variables)
- x_{13} = age, and x_{14} = weight (not repeated measures).

But

x_1 to x_{12} (or x_1 to x_{14}) are *not* repeated measures data because not all the values are comparable.

An example with many variables which is not multivariate

Suppose you have tree data like the following:

- DBH on one set of 100 trees
- height on another set of 100 trees
- age on a third set of 100 trees

These are three *univariate* data sets.

They do *not* make up a multivariate data set.

You have absolutely no information on possible relationships between variables.

Using multivariate methods on such data is like doing a paired t-test with independent samples.

Remark: Paired data is probably the simplest example of multivariate (bivariate) data. In fact, it's repeated measures data.

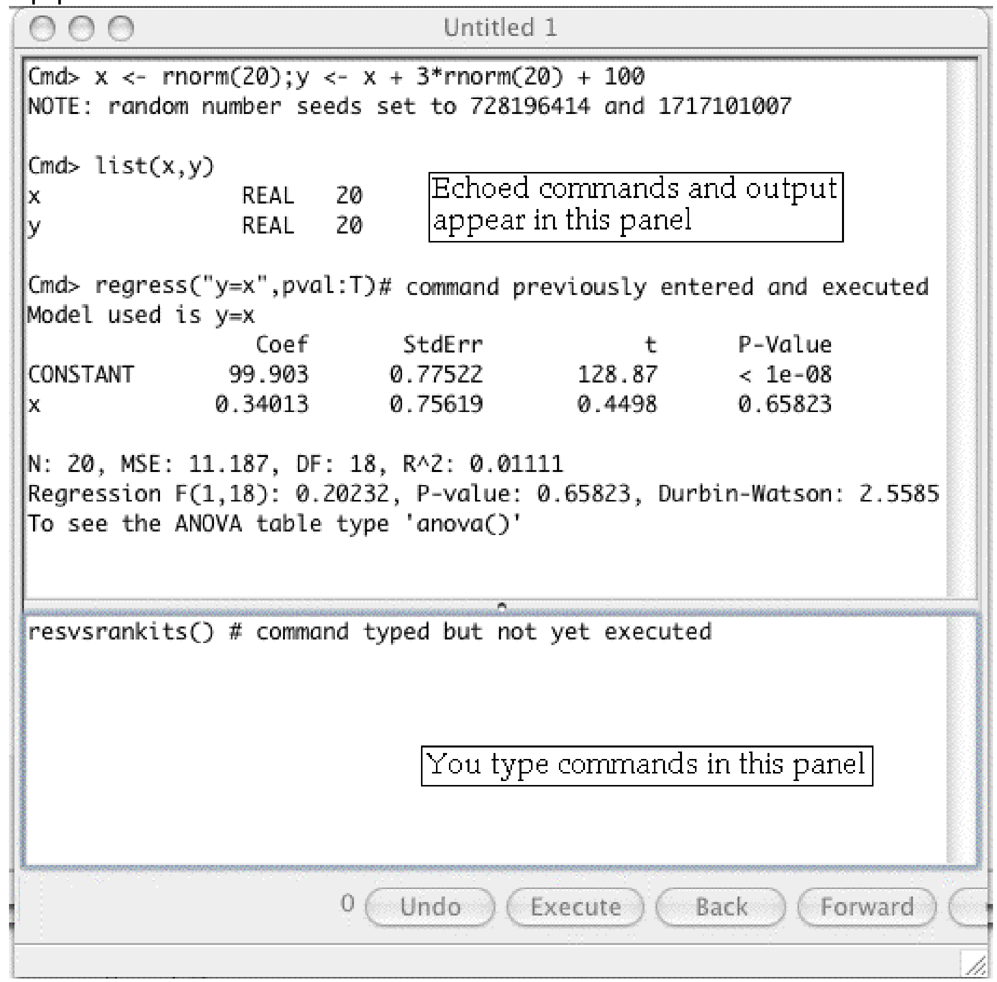
A little bit about MacAnova

There is a completely new version, Carapace MacAnova.

In the older Classic MacAnova, you type commands in a window and output is printed in the same window after the command.

In Carapace MacAnova, you type commands in the **lower panel** of a window with two panels and a row of buttons. Then your command is echoed to the upper panel and is followed by output..

Here is the two panel window as it appears in Macintosh OS X



When you type **Return** or **Enter** after the command it is echoed above with output.

MacAnova as *numerical* calculator

```
Cmd> 3/4          # Cmd> is the "prompt"
(1)          0.75     The answer is automatically printed

Cmd> sqrt(17) + log10(20)  # you can use named functions
(1)        5.4241     4.1231 + 1.301
```

$\sqrt{17} + \log_{10}(20)$

MacAnova as *symbolic* calculator

```
Cmd> pi <- 3*log(640320)/sqrt(163)#natural log

Cmd> # previous lines assigns value to variable pi using

Cmd> # assignment operator <-

Cmd> pi
(1)      3.1416

Cmd> sqrt(2*pi) # square root of 2 times pi
(1)      2.5066
```

Comment starting with "#"

Anything after # is ignored so that you can add comments to any line.

"<-" is the **assignment operator**.

The value of the *right* side is assigned to the variable named on the *left* side.

```
Cmd> PI # predefined variable with value π
(1)        3.1416

Cmd> E # predefined variable with value e
(1)        2.7183
```

Although they have the same *value*, **PI** is a different *variable* from **pi** since upper and lower case matter in names.

Variable names

- Start with letter (a-z, A-Z)
- Continue with letter, number or _
- Length ≤ 12 characters

  x, **residuals**, **Height**, **y1**, **no_treatment**
- Upper and lower *case* matters: **Height** is different from **height**.

No dots in names.

- **pi.hat** is illegal but **pi_hat** is OK.

```
Cmd> pi.hat <- 5/7 # illegal variable name
ERROR:  do not use . in variable names near pi.

Cmd> pi_hat <- 3/7 # legal variable name
```

Names can also start with _ (underscore) but you should avoid such names since they have a special meaning: A variable whose name begins with "_" is "invisible" and you may not see its value when you expect to.

Names can also start with @ followed by a letter (a-z, A-Z).

A variable with such a name is *temporary*; it will be deleted before the next command is executed.

This can be useful, like a scratch pad; you save an intermediate result in a temporary variable, keeping only the final value.

```
Cmd> @tmp <- 3*log(640432); pi <- @tmp/sqrt(163)

Cmd> @tmp
UNDEFINED
```


Assigning a value to a variable

```
Cmd> x <- 3.24
```

"assigns" the value 3.24 to variable x.

- If x already exists, its old value is lost
- If x does not previously exist, it will exist after the assignment.

Seeing the value of a variable

Just typing a variable's name prints its value

```
Cmd> x
(1)       3.24
```

Ignore the number in () for the moment.

You can also use **print()**:

```
Cmd> print(x)
x:
(1)       3.24
```

print(x,y) would print both x and y.

print(x,nsig:12) prints x to 12 significant digits.

See *Introduction to MacAnova* for more examples.

A variable can contain several values:

- A **vector** has 1 dimension
- A **matrix** is a 2 dimensional table
- An **array** has more than 2 dimensions.

```
Cmd> y <- vector(42,52,48,58, 4,5,4,3)

Cmd> # y is a vector made up of all the arguments of vector()

Cmd> y # typing y prints it
(1)          42          52          48          58           4
(6)           5           4           3
```

The numbers, 1 and 6, in () identify the first numbers in the rows as being the first and sixth elements in y.

```
Cmd> x <- matrix(y,4) # make a matrix x with 4 rows and 2 cols

Cmd> # matrix(vec, n) makes a matrix with n rows from vec

Cmd> x   # or print(x); print the value
(1,1)          42           4
(2,1)          52           5
(3,1)          48           4
(4,1)          58           3
```

The pairs of numbers in () identify the first numbers in each row as being elements in rows 1 through 4 and in column 1 of x. For example 48 is in row 3 and column 1.

```
Cmd> x + 5 # You can do arithmetic directly with vector,matrix
(1,1)          47           9
(2,1)          57          10
(3,1)          53           9
(4,1)          63           8
```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You extract information from a vector, matrix or array using **subscripts**.

```
Cmd> y[3] # single number extracts y_3 = element 3 of y
(1)          48

Cmd> y[vector(1,3,5)] # y_1, y-3 and y_5
(1)          42          48           4

Cmd> y[-3] # everything but y_3
(1)          42          52          58           4           5
(6)           4           3

Cmd> run(4) # numbers 1, 2, 3, 4
(1)           1           2           3           4

Cmd> y[-run(4)] # everything but y_1, y_2, y_3, y_4
(1)           4           5           4           3
```

With a *matrix* you need 2 subscripts (row, column)

```
Cmd> x[3,2] # element in row 3 of column 2 of x

Cmd> x[,2] # all of column 2
(1,1)           4
(2,1)           5
(3,1)           4
(4,1)           3
(1,1)           4

Cmd> x[-1, ] # all rows except row 1
(1,1)          52           5
(2,1)          48           4
(3,1)          58           3

Cmd> x[-run(2),2] # column 2 omitting rows 1 and 2
(1,1)           4
(2,1)           3

Cmd> x[x[,1] >= 50,] # rows of x with column 1 >= 50
(1,1)          52           5
(2,1)          58           3
```