Comparing the usual and model based estimates of variance for the ratio estimator

Consider a population where the ratio estimator makes sense. Assuming simple random sampling without replacement we have the usual estimate of its variance. In class I also described the model based approach to the ratio estimate and gave an estimate of variance based on the model.

For $R$ samples the function *compratiolp* calculates the ratio estimate, its absolute error and the two estimates of variance. For each sample it also calculates the sample mean of the $x$ values. The output is a $R$ by five matrix where each row contains the ratio estimate, its absolute error, the usual estimate of variance, the model based estimate of variance and the sample mean of the $x$values. The matrix has been ordered so that the sample with the smallest sample mean of the $x$ values is in the first row. The second row contains the results for the second smallest sample mean of the $x$ values and so on.

```
> ratiototboth<-function(smp,popy,popx)
+ {
+         n <- length(smp)
+         N<-length(popx)
+         ff<-n/N
+         ysamp<-popy[smp]
+         xsamp<-popx[smp]
+         xnsamp<-popx[-smp]
+         tx<-sum(popx)
+         trtot<-sum(popy)
+         rhat <- sum(ysamp)/sum(xsamp)
+         esttot <- rhat * tx
+         err<-abs(esttot - trtot)
+         dum1<-(N*N*(1-ff))/(n*(n-1))
+         usualvartot <- dum1*sum((ysamp-rhat*xsamp)^2)
+         usualans<-c(esttot,err,usualvartot)
+         dum2<-sum(((ysamp -rhat*xsamp)^2/xsamp))*((mean(xnsamp)
+               *mean(popx))/mean(xsamp))
+         modelvartot<-dum1*dum2
+         ans<-c(esttot,err,usualvartot,modelvartot)
+          return(ans)
+ }
> compratiolp<-function(popy,popx,n,R)
+   {
+     N<-length(popy)
+     xbar<-rep(0,R)
+     ans<-matrix(0,R,4)
+     for(i in 1:R){
+       samp<-sample(1:N,n)
+       xbar[i]<-mean(popx[samp])
```

```
+       ans[i,]<-ratiototboth(samp,popy,popx)
+     }
+     xbarord<-order(xbar)
+     xbar<-xbar[xbarord]
+     ans[1:R,]<-ans[xbarord,]
+     ans<-cbind(ans,xbar)
+     ans<-round(ans,digits=2)
+     return(ans)
+   }
> popx<-rgamma(100,5) + 50
> popy<-rnorm(100,2.2*popx,.3*popx)
> out<-compratiolp(popy,popx,10,5)
> out

                                          xbar
[1,] 12711.26 524.35  29894.15  31635.25 53.32
[2,] 12856.22 669.31 161299.18 167142.46 54.06
[3,] 12089.08  97.83 321804.75 327658.39 54.46
[4,] 12430.73 243.83  84560.73  83495.12 54.93
[5,] 11806.37 380.54 153337.59 149500.06 54.98
```