2.) A public opinion researcher has a budget of \$20,000 for taking a survey. She knows that 90% of all households have telephones. Telephone interviews cost \$10 per household; in-person interviews cost \$30 each if all interviews are conducted in person and \$40 each if only nonphone households are interviewed in person (because there will be extra travel costs). Assume that the variances in the phone and nonphone strata are similar and that the fixed costs are $c_0 = \$5000$. How many households should be interviewed in each stratum if

a All households are interviewed in person.

b Households with a phone are contacted by telephone and households without a phone are contacted in person.

ANS.

(a) Because the budget for interviews is \$15,000, a total of $15{,}000/30 = 500$ in-person interviews can be taken. The variances in the phone and nonphone strata are assumed similar, so proportional allocation is optimal: 450 phone households and 50 nonphone households would be selected for interview.

(b) The variances in the two strata are assumed equal, so optimal allocation gives

$$n_h \propto N_h/\sqrt{c_h}.$$

Stratum	c_h	N_h/N	$N_h/(N\sqrt{c_h})$
Phone	10	0.9	0.284605
Nonphone	40	0.1	0.015811
Total		1.0	0.300416

The calculations in the table imply that

$$n_{\text{phone}} = \frac{0.284605}{0.300416}n;$$

the cost constraints imply that

$$10n_{\text{phone}} + 40n_{\text{non}} = 10n_{\text{phone}} + 40(n - n_{\text{phone}}) = 15{,}000.$$

Solving, we have

$$n_{\text{phone}} = 1227$$
$$n_{\text{non}} = 68$$
$$n = 1295.$$

3)

A stratified sample is being designed to estimate the prevalence p of a rare characteristic—say, the proportion of residents in Milwaukee who have Lyme disease. Stratum 1, with N_1 units, has a high prevalence of the characteristic; stratum 2, with N_2 units, has low prevalence. Assume that the cost to sample a unit (for example, the cost to select a person for the sample and determine whether he or she has Lyme disease) is the same for each stratum and that at most 2000 units are to be sampled.

a Let p_1 and p_2 be the respective proportions in stratum 1 and stratum 2 with the rare characteristic. If $p_1 = 0.10$, $p_2 = 0.03$, and $N_1/N = 0.4$, what are n_1 and n_2 under optimal allocation?

b If $p_1 = 0.10$, $p_2 = 0.03$, and $N_1/N = 0.4$, what is $V(\hat{p}_{str})$ under proportional allocation? Under optimal allocation? What is the variance if you take an SRS of 2000 units from the population?

ANS

(a) $n_h = 2000(N_h S_h / \sum_i N_i S_i)$

Stratum	N_h/N	S_h	$S_h N_h/N$	n_h
1	0.4	$\sqrt{(.10)(.90)} = .3000$	.1200	1079
2	0.6	$\sqrt{(.03)(.97)} = .1706$	.1024	921
Total	1.0		.2224	2000

$$S_1^2 = (.10)(.90) = .09 \quad \text{and} \quad S_2^2 = (.03)(.97) = .0291.$$

Under proportional allocation $n_1 = 800$ and $n_2 = 1200$.

$$V_{\text{prop}}(\hat{p}_{\text{str}}) = (.4)^2 \frac{.09}{800} + (.6)^2 \frac{.0291}{1200} = 2.67 \times 10^{-5}.$$

For optimal allocation,

$$V_{\text{opt}}(\hat{p}_{\text{str}}) = (.4)^2 \frac{.09}{1079} + (.6)^2 \frac{.0291}{921} \doteq 2.47 \times 10^{-5}.$$

For an SRS, $p = (.4)(.10) + (.6)(.03) = .058$

$$V_{\text{srs}}(\hat{p}_{\text{srs}}) = \frac{.058(1 - .058)}{2000} = 2.73 \times 10^{-5}.$$