

Forty four people took this exam. The mean score was 75.9 with a standard deviation of 21.5.

1. Let  $X$  be a random variable with cumulative distribution

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.1 & \text{if } 1 \leq x < 5 \\ 0.3 & \text{if } 5 \leq x < 7 \\ 0.7 & \text{if } 7 \leq x < 9 \\ 0.9 & \text{if } 9 \leq x < 10 \\ 1 & \text{if } x \geq 10 \end{cases}$$

i) Find  $P(X \geq 6)$ .

ii) Find  $E(X)$ .

**Solution:**

$$P(X \geq 6) = P(X = 7) + P(X = 9) + P(X = 10) = 0.4 + 0.2 + 0.1$$

$$E(X) = 1 \times 0.1 + 5 \times 0.2 + 7 \times 0.4 + 9 \times 0.2 + 10 \times 0.1$$

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2. One in every fourteen undergraduate students at the University of Minnesota is infected by a certain virus. A medical researcher begins to select students at random and examines them for the virus.

i) What is the probability that exactly two of the first nine students the researcher examines will be infected with the virus?

ii) What is the probability that the researcher finds eight students without the virus before she discovers one who is infected.

**Solution:**

i)

$$\binom{9}{2} \left(\frac{1}{14}\right)^2 \left(\frac{13}{14}\right)^7$$

ii)

$$\left(\frac{13}{14}\right)^8 \left(\frac{1}{14}\right)^1$$

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3. A random sample of size three from a normal population with unknown mean and variance yielded the sample values 22.8, 32.8 and 27.8. Find a 99% confidence interval for the population mean.

**Solution:**

Note  $\bar{x} = 27.8$  and  $s^2 = 25$  and since  $t_{2,.005} = 9.925$  we have

$$27.8 \mp \frac{5}{\sqrt{3}} 9.925$$

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4. A random sample of 120 senior citizens yielded 43 who had download music from off the net.  
 i) At level  $\alpha = 0.01$  test the hypothesis that the proportion of senior citizens who have downloaded music from off the net is less than or equal to 30% against the alternative the proportion is greater than 30%.

ii) Find the  $p$ -value for these data.

iii) Find a 99% confidence interval for the proportion of senior citizens who have downloaded music from off the net.

**Solution:**

i) Since

$$\frac{43/120 - .30}{\sqrt{(.3 \times .7)/120}} = \frac{0.05833}{0.04183} = 1.394 < 2.326 = z_{.01}$$

we accept the null hypothesis.

ii) Since if the null hypothesis is true  $X$ , the number of senior citizens who have downloaded music from off the net, is binomial(120,.3) or approximately normal(36,25.2) we must find the value of  $\alpha$  which satisfies the equation

$$\frac{42.5 - 36}{\sqrt{25.2}} = 1.295 = z_{\alpha}$$

or the  $p$ -value= $P(N(0,1) > 1.295) = 0.0977$ . Note the above uses the continuity correction for the normal approximation to the binomial distribution.

iii) Since  $z_{.005} = 2.576$  we have

$$\frac{43}{120} \mp 2.576 \sqrt{(43/120)(1 - 43/120)/120}$$

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5. A small brewery has two bottling machines. Machine A produces 70% of the bottles and Machine B produces 30%. One out of every 25 bottles filled by A is rejected for some reason, while 1 out of every 20 bottles from B is rejected.

i) What is the probability that a randomly selected bottle is rejected?

ii) What is the probability that a randomly selected bottle is from Machine A, given that it is rejected?

**Solution:**

i)

$$\begin{aligned} P(R) &= P(R \cap A) + P(R \cap B) \\ &= P(A)P(R|A) + P(B)P(R|B) \\ &= 0.7 \times 0.04 + 0.3 \times 0.05 \end{aligned}$$

ii)

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{0.7 \times 0.04}{P(R)}$$

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