Stat 5021, First exam, Spring '04 Name ________________

Forty four people took the exam. There were 9 scores in the 90's, 9 in the 80's, 9 in the 70's, 5 in the 60's, 6 in the 50's, 2 in the 40's, 2 in the 30's and 2 in the 20's.

1. Let the joint distribution of X and Y be given in the table below.

		Y			
		1	2	3	4
X	0	.15	.10	.10	.20
	1	.05	.25	.10	.05

i) Find the probability distribution of Y.
ii) Find the expected value of Y.
iii) Find the covariance of X and Y.
Solution: i) Y takes on the values 1, 2, 3 and 4 with probabilities .20, .35, .20 and .25.
ii)

$$E(Y) = \sum_y yP(Y = y) = 1 \times .20 + 2 \times .35 + 3 \times .20 + 4 \times .25 = 2.5$$

iii) Now $Cov(X, Y) = E(XY) - E(X)E(Y)$ and since $E(X) = .45$ we need to find

$$E(XY) = \sum_{(x,y)} xyP(X = x, Y = y) = 1 \times .050 + 2 \times .25 + 3 \times .10 + 4 \times .05 = 1.05$$

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2. Suppose an urn contains three blue and two white balls. Consider the random experiment where a ball is selected at random from the urn. The color of the ball selected is noted and then the ball is returned to the urn along with an additional ball of the same color. The urn now contains six balls. Another ball is now selected at random from the urn and its color is noted.

i) Find the sample space for this experiment.

ii) Let A be the event that the first ball is blue and B be the event that the second ball is blue. Are A and B independent? Justify your answer.
Solution:
i)

Point	Probability
B_1B_2	(3/5)(4/6)
B_1W_2	(3/5)(2/6)
W_1B_2	(2/5)(3/6)
W_1W_2	(2/5)(3/6)

ii) Note

$$P(A) = P(B_1B_2) + P(B_1W_2) = 12/30 + 6/30 = 18/30, \; P(B) = 18/30 \quad \text{and} \quad P(A \cap B) = 12/30.$$

The two events are dependent since

$$PA \cap B) \neq P(A)P(B)$$

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3. In a random sample of 15 distance runners the average resting heart rate was 58.3 beats/minute with a standard deviation of 5.7 beats/minute. Assume that the resting heart rates of distance runners are normally distributed. i) Find a 95% confidence interval for μ, the true mean resting heart rate for the population of all distance runners.

ii) Suppose another 95% confidence interval for μ is to be calculated but this time using a random sample of 40 distance runners. In what respects would this interval likely to be different from the interval of part i)?

Solution:

i) Since $t_{14,.025} = 2.145$ we have

$$58.3 \mp \frac{5.7}{\sqrt{15}} 2.145$$

ii) Assuming the sample mean, $\overline{x}$, and sample standard deviation, s of the second sample are roughly equal to those in the first sample we would expect the interval to be shorter. Recall the t-interval for this problem for a a sample of size n is given by

$$\overline{x} \mp \frac{s}{\sqrt{n}} t_{n-1,.025}$$

So as n gets larger the second term in the above will tend to get smaller because of the $\sqrt{n}$ in the denominator and because $t_{n-1,.025}$ gets smaller.

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4. Let $X_1, \ldots, X_{12}$ be independent and identically distributed each Normal$(\mu, 17^2)$. Consider testing the hypothesis H: $\mu = 50$ against the alternative K: $\mu > 50$. Suppose we decide to reject H if and only if $\bar{X} > 58.76$.

i) Find the probability of the Type I error for this test.

ii) Find the probability of the Type II error for this test when $\mu = 58$.

iii) For $\mu > 50$ let $\beta(\mu)$ be the probability of making the Type II error when μ is the true mean. Make a rough sketch of the graph of $\beta(\mu)$ for $\mu > 50$.

Solution:

i)

$$\begin{aligned} P_{\mu=50}(\bar{X} > 58.76) &= P\left(N(0,1) > (58.76 - 50)/\frac{17}{\sqrt{12}}\right) \\ &= P\left(N(0,1) > \frac{8.76}{4.907}\right) = P(N(0,1) > 1.785) = .0371 \end{aligned}$$

ii)

$$\begin{aligned} P_{\mu=58}(\bar{X} < 58.76) &= P\left(N(0,1) < (58.76 - 58)/\frac{17}{\sqrt{12}}\right) \\ &= P\left(N(0,1) < \frac{0.88}{4.907}\right) = P(N(0,1) < 0.155) = .562 \end{aligned}$$

iii) Note that you just need to compute a few more values as in part ii) to make the plot.

μ	$\beta(\mu)$
50	.9629
52	.916
55	.778
58	.562
61	.324
64	.143
67	.047

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