Stat 5021, First exam, Fall '04 Name _____

Forty three people took the exam. The range of the scores was from 36 to 100 with a median of 79. The mean was 74.2 and the standard deviation was 18.9.

1. Suppose a random variable X takes on the values -3, 0 and 3 with probabilities 3/7, 1/7 and 3/7 respectively.

i) Find the variance of X.

ii) Find the expected value of $2X^3$.

Solution:

i)

$$E(X) = (-3) \times \frac{3}{7} + 0 \times \frac{1}{7} + 3 \times \frac{3}{7} = 0$$

$$V(X) = E(X^2) - (E(X))^2 = E(X^2) = (-3)^2 \times \frac{3}{7} + 0 \times \frac{1}{7} + 3^2 \times \frac{3}{7} = \frac{54}{7}$$

ii)

$$E(2X^3) = 2E(X^3) = 2\left\{(-3)^3 \times \frac{3}{7} + 0 \times \frac{1}{7} + 3^3 \times \frac{3}{7}\right\} = 0$$

∎

2. An urn contains twelve balls numbered 1 to 12. Using simple random sampling with replacement five balls are selected from the urn.

i) What is the expected number of times that ball 1 will appear?

ii) Find the probability that ball 1 does not appear in the five selections.

iii) Find the probability that at least one of the two balls numbered 1 or 2 does not appear in the five selections.

Solution:

i)

$$5 \times \frac{1}{12}$$

ii)

$$\left(\frac{11}{12}\right)^5$$

iii) Let A_i be the event that ball i does not appear in the five selections. Then what we want is

$$\begin{aligned} P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= \left(\frac{11}{12}\right)^5 + \left(\frac{11}{12}\right)^5 - \left(\frac{10}{12}\right)^5 \end{aligned}$$

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3. A random sample of 70 adults yielded 12 who regularly watched the TV show Stupid Videos.

i) Find a 99% confidence interval for p the proportion of adults who regularly watch Stupid Videos.

ii) If we were testing H:$p = 0.15$ against K:$p > 0.15$ what is the level of significance or p-value of the above data.

Solution:

i) Note $\hat{p} = 12/70$ and $z_{.005} = 2.576$ and so

$$\frac{12}{70} \mp \sqrt{\frac{\frac{12}{70}\frac{58}{70}}{70}} 2.576$$

ii)

$$\frac{\frac{12}{70} - 0.15}{\sqrt{\frac{0.15 \times 0.85}{70}}} = \frac{0.0214}{0.0427} = 0.501$$

The $p - value = P(\text{Normal}(0,1) > 0.501) = 0.308$.

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4. A random sample of size 11 was taken from a normal population with unknown mean μ and unknown variance σ^2. If the observed data yielded a sample mean of 15.45 and a sample variance of 22.36 find a 99% confidence interval for μ.

Solution:

Since $P(T_{10} > 3.169) = .005$ the interval is

$$\left(15.45 - 3.169\frac{\sqrt{22.36}}{\sqrt{11}}\,,\; 15.45 + 3.169\frac{\sqrt{22.36}}{\sqrt{11}}\right)$$

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5. A random sample of size 9 was taken from a normal population with unknown mean μ and known variance 12.5.

i) For testing $H : \mu = 5$ against the alternative $K : \mu \neq 5$ give the usual critical or rejection region for the null hypothesis at level $\alpha = 0.05$.

ii) Find the probability of making the Type II error when $\mu = 7$.

Solution:

i) Sine $z_{0.025} = 1.96$ we will reject H when

$$\left| \frac{\bar{X}_9 - 5}{\sqrt{12.5}/3} \right| > 1.96$$

Or when $\bar{X}_9 < 2.69$ or $\bar{X}_9 > 7.31$.

ii) Since $\sqrt{12.5}/3 = 1.18$

$$P(2.69 < N(7, 12.5/9) < 7.31) = P(\frac{2.69 - 7}{1.18} < N(0,1) < \frac{7.31 - 7}{1.18}) = .60 - 0 = 0.60$$

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