

HW 2

1. Suppose an urn contains three blue and two white balls. Consider the random experiment where a ball is selected at random from the urn. The color of the ball selected is noted and then the ball is returned to the urn along with an additional ball of the same color. The urn now contains six balls. Another ball is now selected at random from the urn and its color is noted.

i) Find the sample space for this experiment.

ii) Let A be the event that the first ball is blue and B be the event that the second ball is blue. Are A and B independent? Justify your answer.

iii) Let X be the total number of blue balls selected. Find $E(X)$, the expected value of X .

2. Suppose urn I contains five balls with two of them labeled zero, one labeled one, one labeled two and one labeled three. Suppose urn II also contains five balls with one labeled two, two labeled three and two labeled four. Suppose a ball is selected at random from each urn. Let A be the event that the two balls selected have the same value. Let B be the event that the ball selected from urn II is larger than the ball selected from urn I . Let C be the event that the ball selected from urn II is labeled four.

i) Find $P(A)$.

ii) Find $P(C|B)$.

3. Suppose a random variable X takes on the values -3 , 0 and 3 with probabilities $3/7$, $1/7$ and $3/7$ respectively.

i) Find the variance of X .

ii) Find the expected value of $2X^3$.

4. Let the joint distribution of X and Y be given in the table below.

		Y			
		1	2	3	4
X	0	.15	.10	.10	.20
	1	.05	.25	.10	.05

i) Find the probability distribution of Y .

ii) Find the expected value of Y .

iii) Find the covariance of X and Y .

iv) Find $P(Y > 2 | X=0)$

5. Let X be a random variable with cumulative distribution

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.1 & \text{if } 1 \leq x < 5 \\ 0.3 & \text{if } 5 \leq x < 7 \\ 0.7 & \text{if } 7 \leq x < 9 \\ 0.9 & \text{if } \leq x < 10 \\ 1 & \text{if } x \geq 10 \end{cases}$$

- i) Find $P(X \geq 6)$.
 - ii) Find $E(X)$.
6. Exhibit a discrete random variable which has a variance of 8.5. That is list its set of possible values along with the corresponding probabilities.
7. Suppose an urn contains 4 balls labeled 0, 1, 2 and 4.
- i) If two balls are drawn at random from the urn without replacement let X be their sum. Find the probability distribution of X ,
 - ii) Repeat but now assume that the sampling was done with replacement.
8. Suppose on any given day the probability that Sam receives at least one junk phone call, i.e. a call from some one trying to sell him something, is 0.5 and the probability that he receives at least one junk email is 0.4. Assume that on any given day these events are independent and independent from day to day.
- i) What is the probability that Sam will receive at least one junk phone call and at least one junk email today?
 - ii) In the next ten days how many days should Sam expect to receive at least one junk phone call and at least one junk email during the day?
9. A Master of Ceremonies (MC) has a box and two urns labeled I and II . In the box there two slips labeled I and three slips labeled II . In Urn I there are three balls labeled two and six balls labeled four. In Urn II there are seven balls labeled two and five balls labeled four. The MC selects a slip at random from the box. If it is labeled I then the MC selects two balls at random with replacement from urn I . If the slip is labeled II then MC does the same from urn II . Let X be the sum of the two balls selected.
- i) Find the probability distribution of X .
 - ii) Find the conditional probability that urn I was selected given the event that $X = 4$ was observed.
10. A small brewery has two bottling machines. Machine A produces 70% of the bottles and Machine B produces 30%. One out of every 25 bottles filled by A is rejected for some reason, while 1 out of every 20 bottles from B is rejected.
- i) What is the probability that a randomly selected bottle is rejected?
 - ii) What is the probability that a randomly selected bottle is from Machine A, given that it is rejected?