Stat 5021 Midterm Exam, Fall '15 Name ________________________

You may use one 8" by 11" formula sheet (both sides) but you **may not use** any electronic computing device. You **do not need** to reduce numerical formulas to their simplest form. Your answers may contain R commands.

Sixty four people took the exam. There were 23 in the 90's with four scores of 100. There were 17 in the 80's, 8 in the 70's, 6 in the 60's, 2 in the 50's and 4 below 50.

1. Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.1 & \text{if } 1 \leq x < 4 \\ 0.4 & \text{if } 4 \leq x < 6 \\ 0.7 & \text{if } 6 \leq x < 9 \\ 0.9 & \text{if } 9 \leq x < 11 \\ 1 & \text{if } x \geq 11 \end{cases}$$

Find $E(X)$.

Solution:

$$E(X) = 1 \times 0.1 + 4 \times 0.3 + 6 \times 0.3 + 9 \times 0.2 + 11 \times 0.110 \times 0.1$$

∎

2. A small brewery has two bottling machines. Machine A produces 70% of the bottles and Machine B produces 30%. One out of every 25 bottles filled by A is rejected for some reason, while 1 out of every 20 bottles from B is rejected.

i) What is the probability that a randomly selected bottle is rejected?

ii) What is the probability that a randomly selected bottle is from Machine A, given that it is rejected? ***Solution:***

i)

$$\begin{aligned} P(R) &= P(R \cap A) + P(R \cap B) \\ &= P(A)P(R|A) + P(B)P(R|B) \\ &= 0.7 \times 0.04 + 0.3 \times 0.05 \end{aligned}$$

ii)

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{0.7 \times 0.04}{P(R)}$$

∎

3. Let X be a Binomial$(60, p)$ random variable. For testing H: $p = 0.6$ against K: $p \neq 0.6$ consider the test which rejects H if and only if $X = 0, 1, \ldots, 28$ or $X = 45, 46, \ldots, 60$. For both parts of this question your answer should include both a mathematical expression and a R command.

i) For this test find α, the probability of the type I error.

ii) For this test, if $p = 0.8$, find β, the probability of the type II error.

Solution:

i)

$$\sum_{x=0}^{28} \binom{60}{x} 0.6^x 0.4^x + \sum_{x=45}^{60} \binom{60}{x} 0.6^x 0.4^{60-x} = pbinom(60, 28, .6) + 1 - pbinom(60, 44, .6)$$

ii)

$$\sum_{x=29}^{44} \binom{60}{x} 0.8^x 0.2^{60-x} = pbinom(60, 44, 0.8) - pbinom(60, 28, 0.8)$$

∎

4. A study of the relationship between level of service (Poor or Good) and aggressiveness in pricing for a variety of stores reported the accompanying data based on a sample of 200. At level $\alpha = .01$ test the hypothesis that facility conditions and pricing policy are independent of one another.

	Aggressive	Non-aggressive
Poor	45	45
Good	15	95

Solution: Under the null hypothesis the expected number in the poor by aggressive cell is $200 \frac{90}{200} \frac{60}{200} = 27$. Finding the expected number for the other cells in the same way we have that

$$\sum \frac{(O-E)^2}{E} = \frac{(45-27)^2}{27} + \frac{(45-63)^2}{63} + \frac{(15-33)^2}{33} + \frac{(95-77)^2}{77}$$
$$> 6.63490 = qgamma(.99, .5, .5) = \chi^2_{1,.01}$$

and so we reject the hypothesis of independence. ∎

5. A random sample of size 11 was taken from a normal population with unknown mean μ and unknown variance σ^2. If the observed data yielded a sample mean of 15.45 and a sample variance of 22.36 give a mathematical expression for the 99% confidence interval for μ.

Solution:

Since $qt(0.995, 10) = 3.169$ or $P(T_{10} > 3.169) = .005$ the interval is

$$\left(22.36 - 3.169 \frac{\sqrt{22.36}}{\sqrt{11}} \, , \, 22.36 + 3.169 \frac{\sqrt{22.36}}{\sqrt{11}} \right)$$

∎

6. A study was conducted to investigate the relationship between self esteem and whether or not a person had a heart attack within the past two years. A random sample of 170 individuals who had a heart attack within the past two years yielded 75 with high self esteem while an independent random sample of 190 individuals who had not suffered a heart attack yielded 103 with high self esteem. Let p_1 and p_2 be the proportions of people with high self esteem in the populations of people who have had heart attacks and those who have not. Using the given data find the value of the unbiased estimate of $p_1 - p_2$. Give the standard error of your estimate.

Solution: Let $\hat{p}_1 = 75/170$ and $\hat{p}_2 = 103/190$ then the estimate is $\hat{p}_1 - \hat{p}_2$.

If X is binomial(n, p) then the variance of X/n is $(p(1-p))/n$. Since the standard error of an estimate is its estimated standard deviation we see that the standard error in question is

$$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

∎

7. Suppose we have three urns, the first contains 2 blue and 5 white balls, the second 7 blue and 4 white and the third 12 blue and 4 white. Consider the experiment where we independently draw one ball at random from each urn.

i) Find the probability that a blue ball selected is from the second urn and white balls from the other two.

ii) Find the probability that at least one of the three balls selected is blue.

Solution:

i)

$$\frac{5}{7}\frac{7}{11}\frac{4}{16}$$

ii)

$$1 - \frac{5}{7}\frac{4}{11}\frac{4}{16}$$

∎