Stat 5021 Midterm Exam, Fall ‘14 Name ____________________

Fifty three people took the exam. There was 1 in the 90’s, 4 in the 80’s, 9 in the 70’s, 9 in the 60’s, 14 in the 50’s, 6 in the 40’s, 7 in the 30’s and 3 in the 20’s.

1. An urn contains twelve balls numbered 1 to 12. Using simple random sampling with replacement five balls are selected from the urn.

i) What is the expected number of times that ball 1 will appear?

ii) Find the probability that ball 1 does not appear in the five selections.

iii) Find the probability that at least one of the two balls numbered 1 and 2 does not appear in the five selections.

Ans i)

$$5 \times \frac{1}{12}$$

ii)

$$\left(\frac{11}{12}\right)^5$$

iii) Let A_i be the event that ball i does not appear in the five selections. Then what we want is

$$\begin{aligned} P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= \left(\frac{11}{12}\right)^5 + \left(\frac{11}{12}\right)^5 - \left(\frac{10}{12}\right)^5 \end{aligned}$$

2. Let X be a random variable with cumulative distribution

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.1 & \text{if } 1 \le x < 3 \\ 0.3 & \text{if } 3 \le x < 5 \\ 0.9 & \text{if } 5 \le x < 7 \\ 1 & \text{if } x \ge 7 \end{cases}$$

Find the variance of the random variable $Y = 2X + 12$.

Ans Now

$$E(X) = 1 \times 0.1 + 3 \times 0.2 + 5 \times 0.6 + 7 \times 0.1 = 4.4$$

$$E(X^2) = 1 \times 0.1 + 9 \times 0.2 + 25 \times 0.6 + 49 \times 0.1 = 21.8$$

and so $V(X) = E(X^2) - (E(X))^2 = 21.8 - 19.36 = 2.44$ and $V(Y) = 4V(X) = 4 \times 2.44$.

3. Let $X_1, \ldots, X_6$ be iid normal(μ,9).

i) For testing $H : \mu \le 30$ against $K : \mu > 30$ state the usual level $\alpha = .01$ test for this problem and find the probability of making the type II error for this test when $\mu = 34$.

ii) If $\bar{x} = 31.72$ find the p-value for this outcome.
Ans
i) Since $qnorm(.99) = z_{.01} = 2.33$ we reject H if and only if

$$\frac{\bar{X} - 30}{3/\sqrt{6}} > 2.33 \quad \text{or when} \quad \bar{X} > 32.84 = c$$

and so

$$P_{\mu=34}(\bar{X} < c) = pnorm(c, 34, 3/\sqrt{6})) = 0.1718$$

ii) Since

$$\frac{31.72 - 30}{3/\sqrt{6}} = 1.40 = z_\alpha$$

we have $\alpha = 0.08 = 1 - pnorm(1.4)$.

4. In a random sample of 15 distance runners the average resting heart rate was 58.3 beats/minute with a standard deviation of 5.7 beats/minute. Assume that the resting heart rates of distance runners are normally distributed.

i) Find a 95% confidence interval for μ, the true mean resting heart rate for the population of all distance runners.

ii) Suppose another 95% confidence interval for μ is to be calculated but this time using a random sample of 40 distance runners. In what respects would this interval likely to be different from the interval of part i)?

Ans i) Since $qt(.975, 14) = t_{14,.025} = 2.145$ we have

$$58.3 \mp \frac{5.7}{\sqrt{15}} 2.145$$

ii) Assuming the sample mean, $\overline{x}$, and sample standard deviation, s of the second sample are roughly equal to those in the first sample we would expect the interval to be shorter. Recall the t-interval for this problem for a a sample of size n is given by

$$\overline{x} \mp \frac{s}{\sqrt{n}} t_{n-1,.025}$$

So as n gets larger the second term in the above will tend to get smaller because of the $\sqrt{n}$ in the denominator and because $t_{n-1,.025}$ gets smaller.

5. 95% of the widgets produced by supplier I pass inspection while 90% of those produced by supplier II pass inspection. Suppose 60% of the shipments come from supplier I while the remaining 40% come from supplier II.

i) Find an expression for the probability that in a random shipment containing 90 widgets at least 80 of them will pass inspection.

ii) Given that in a shipment of 90 widgets at least 80 of them passed inspection find an expression for the conditional probability that the shipment came from supplier I.

Ans i) Let A be the event that at least 80 in a shipment of 90 pass inspection then

$$P(A) = P(I)P(A|I) + P(II)P(A|II) = 0.60p_1 + 0.40p_2$$

where

$$p_1 = P(A|I) = \sum_{x=80}^{90} \binom{90}{x} (.95)^x (.05)^{9-x} = 1 - pbinom(79, 90, .95) = 0.9947071$$

$$p_2 = P(A|II) = \sum_{x=80}^{90} \binom{90}{x} (.9)^x (.1)^{90-x} = 1 - pbinom(79, 90, .90) = 0.7125117$$

ii) By Bayes theorem

$$P(I|A) = \frac{P(I \cap A)}{P(A)} = \frac{P(I)P(A|I)}{P(A)} = \frac{0.60p_1}{0.60p_1 + 0.40p_2}$$