Consider a metallurgical project that involved the study of the tempering response of a certain grade of steel. Slugs of this steel were preprocessed to reasonably uniform hardness which was measured and recorded. The slugs were then tempered at various temperatures for various lengths of time. The hardness was then measured and the the change in hardness, Y, computed. (Note a negative value for Y means that the tempering process has made the steel harder.)

There were four different lengths of time 5, 50, 150 and 500 minutes and four different temperatures 800, 900, 1000 and 1100 degrees Fahrenheit. There were two independent measurements taken at each of the $4 \times 4 = 16$ possible combinations. First the quadratic model

$$Y = \beta_0 + \beta_1 \ln(X_1) + \beta_2 X_2 + \beta_3 (\ln(X_1))^2 + \beta_4 X_2^2 + \beta_5 X_2 \ln(X_1) + Z$$

was fit to the data where X_1 was time and X_2 was temperature.

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.932e+01  2.684e+01   1.838 0.077588 .
ln(X1)       9.363e+00  1.747e+00   5.359 1.31e-05 ***
X2          -1.235e-01  5.586e-02  -2.211 0.036010 *
ln(X1)sq    -5.252e-01  1.230e-01  -4.269 0.000231 ***
X2sq         6.875e-05  2.917e-05   2.357 0.026249 *
ln(X1):X2   -6.533e-03  1.538e-03  -4.247 0.000245 ***
---
Signif. codes:  0  '***'  0.001  '**'  0.01  '*'  0.05  '.'  0.1  ' '  1

Residual standard error: 1.65 on 26 degrees of freedom
Multiple R-Squared: 0.8304,     Adjusted R-squared: 0.7978
F-statistic: 25.46 on 5 and 26 degrees of freedom,      p-value: 3.002e-09

Analysis of Variance Table

Response: y
          Df  Sum Sq Mean Sq F value    Pr(>F)
ln(X1)     1  68.813  68.813 25.2693 3.131e-05 ***
X2         1 164.025 164.025 60.2332 3.109e-08 ***
ln(X1)sq   1  49.620  49.620 18.2216 0.0002313 ***
X2sq       1  15.125  15.125  5.5542 0.0262487 *
ln(X1):X2  1  49.115  49.115 18.0359 0.0002450 ***
Residuals 26  70.802   2.723
```

Next the standard two way anova model was fit to these data. Below is the anova table, the individual cell means along with a plot of the cell means.

```
> hard.aov_aov(y~Time*Temp)
> summary(hard.aov)
           Df  Sum Sq Mean Sq F value     Pr(>F)
Time        3 119.250  39.750 11.3571  0.000307 ***
Temp        3 182.750  60.917 17.4048 2.721e-05 ***
Time:Temp   9  59.500   6.611  1.8889  0.127969
Residuals  16  56.000   3.500

> sapply(split(y,interaction(Time,Temp)),mean)
  1.1   2.1   3.1   4.1
 -0.5   3.5   3.0  -1.0

  1.2   2.2   3.2   4.2
 -2.5  -2.0  -1.5  -5.0

  1.3   2.3   3.3   4.3
 -1.0  -2.5  -4.5  -7.5

  1.4   2.4   3.4   4.4
 -1.5  -3.0  -6.0 -10.0
```