A brief introduction to fuzzy set theory

This is a quick introduction to some of the basic concepts and terminology of fuzzy set theory, which can be found in the most elementary of introductions to the subject. See for example *Fuzzy Set Theory: Foundations and Applications* by Klir, St. Clair, and Yuan. Published by Prentice Hall PTR in 1997.

A *fuzzy set* A in a space Θ is characterized by its *membership function*, which is a map $I_A : \Theta \to [0, 1]$. The value $I_A(\theta)$ is the "degree of membership" of the point θ in the fuzzy set A or the "degree of compatibility ... with the concept represented by the fuzzy set". See page 75 of (Klir, St. Clair, and Yuan, 1997). The idea is that we are uncertain about whether θ is in or out of the set A. The value $I_A(\theta)$ represents how much we think θ is in the fuzzy set A. The closer $I_A(\theta)$ is to 1.0, the more we think θ is in A. The closer $I_A(\theta)$ is to 0.0, the more we think θ is not in A.

A fuzzy set whose membership function only takes on the values zero or one is called *crisp*. For a crisp set, the membership function I_A is the same thing as the indicator function of an ordinary set A. Thus "crisp" is just the fuzzy set theory way of saying "ordinary," and "membership function" is the fuzzy set theory way of saying "indicator function." The *complement* of a fuzzy set A having membership function I_A is the fuzzy set B having membership function $I_B = 1 - I_A$.

If I_A is the membership function of a fuzzy set A, the γ-*cut* of A (Klir, St. Clair, and Yuan, 1997, Section 5.1) is the crisp set

$$ {}^{\gamma}I_A = \{\theta : I_A(\theta) \geq \gamma\}. $$

Clearly, knowing all the γ-cuts for $0 \leq \gamma \leq 1$ tells us everything there is to know about the fuzzy set A. The 1-cut is also called the *core* of A, denoted $\operatorname{core}(A)$ and the set

$$ \operatorname{supp}(A) = \bigcup_{\gamma > 0} {}^{\gamma}I_A = \{\theta : I_A(\theta) > 0\} $$

is called the *support* of A (Klir, St. Clair, and Yuan, 1997, p. 100). A fuzzy set is said to be *convex* if each γ-cut is convex (Klir, St. Clair, and Yuan, 1997, pp. 104–105).