Statistics 5401
23. Factor Rotation (addendum)

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```
Cmd> R <- cor(o2)

Cmd> out <- extractml(R,2)$L
(1,1)      0.87422            0
(2,1)      0.84993      -0.5069
(3,1)      0.49093      0.82364
(4,1)      0.53638      0.61417
```

This has one real 0, but not many small numbers.
Consider rotation by the matrix

$$\mathbf{H} = \begin{bmatrix} 0.87651 & 0.48139 \\ -0.48139 & 0.87651 \end{bmatrix}$$

Then we get

```
Cmd> L %*% H
(1,1)      0.76626      0.42084
(2,1)      0.98898    -0.035163
(3,1)     0.033813      0.95825
(4,1)      0.17449      0.79653
```

No exact zeroes, but more smallish loadings
Begin with the O2 data.

```
Cmd> L<- facanal(R,2)$loadings;L
(1,1)      0.56905      0.74273
(2,1)      0.13354      0.99104
(3,1)      0.99892     -0.04657
(4,1)      0.83687      0.12391

Cmd> rotation(L,method:"varimax")
(1,1)       0.4683      0.81004
(2,1)    0.0043494      0.99999
(3,1)      0.99656      0.08291
(4,1)      0.81384      0.23102

Cmd> rotation(L,method:"varimax",kaiser:T)
(1,1)      0.47425      0.80657
(2,1)     0.011718      0.99993
(3,1)      0.99714     0.075565
(4,1)      0.81552      0.22501
```

```
Cmd> rotation(L,method:"equimax")
(1,1)       0.4683      0.81004
(2,1)    0.0043494      0.99999
(3,1)      0.99656      0.08291
(4,1)      0.81384      0.23102

Cmd> rotation(L,method:"quartimax")
(1,1)      0.47996      0.80319
(2,1)     0.018807      0.99982
(3,1)      0.99765     0.068493
(4,1)      0.81709      0.21922

Cmd> R <- cor(X)

Cmd> L <- facanal(R,2,silent:T)$loadings;L
(1,1)      0.73082     -0.62041
(2,1)      0.79165     -0.54542
(3,1)      0.85494     -0.34299
(4,1)      0.91585     -0.16087
(5,1)      0.95793    -0.025686
(6,1)      0.97239      0.14375
(7,1)      0.98062      0.14294
(8,1)      0.92291       0.2495

Cmd> rotation(L)
(1,1)      0.30043     -0.91036
(2,1)      0.39142     -0.87805
(3,1)      0.55093     -0.73827
(4,1)      0.69783     -0.61458
(5,1)      0.80418     -0.52113
(6,1)      0.90482     -0.38406
(7,1)      0.91142     -0.38904
(8,1)       0.9177     -0.26801

Cmd> rotation(L,method:"quartimax")
(1,1)      0.75738     -0.58769
(2,1)      0.81485      -0.5101
(3,1)      0.86919     -0.30508
(4,1)      0.92204     -0.12046
(5,1)      0.95813     0.016438
(6,1)      0.96513      0.18634
(7,1)      0.97339      0.18589
(8,1)      0.91105      0.28982

Cmd> R <- cor(cars)
```

```
Cmd> L <- facanal(R,10,silent:T)$loadings

Cmd> L2 <- rotation(L)

Cmd> print(L2,format:"f5.2")
L2:
 (1,1)  0.21 -0.89 -0.16  0.21  0.28  0.00 -0.03  0.13 -0.02 -0.02
 (2,1)  0.19 -0.95 -0.10  0.00  0.22 -0.00 -0.03  0.06 -0.02  0.01
 (3,1)  0.17 -0.95 -0.06 -0.16  0.17 -0.01 -0.02 -0.00 -0.02  0.03
 (4,1) -0.81  0.34  0.15  0.00 -0.42 -0.10 -0.00 -0.07 -0.13 -0.01
 (5,1) -0.81  0.33  0.15 -0.03 -0.35 -0.10  0.04  0.11 -0.03  0.02
 (6,1) -0.00 -0.54  0.02 -0.01  0.28  0.02  0.11  0.15  0.15  0.24
 (7,1)  0.14  0.29 -0.11  0.00 -0.21 -0.16 -0.05 -0.61 -0.01  0.00
 (8,1)  0.25 -0.46 -0.14  0.14  0.68  0.15 -0.03  0.01  0.13 -0.15
 (9,1)  0.26 -0.38 -0.22  0.04  0.77  0.26 -0.02  0.13  0.15 -0.18
(10,1)  0.27 -0.61  0.00  0.04  0.67 -0.25  0.15 -0.05  0.02 -0.10
(11,1) -0.19 -0.11  0.21 -0.00 -0.33 -0.79  0.04 -0.07 -0.17 -0.01
(12,1) -0.33  0.19  0.14  0.01 -0.66 -0.23 -0.01 -0.12 -0.29  0.07
(13,1) -0.09  0.21  0.31 -0.10 -0.53 -0.17  0.36  0.04 -0.35  0.05
(14,1)  0.49 -0.36 -0.28 -0.08  0.61  0.10 -0.09 -0.03 -0.14  0.01
(15,1)  0.38  0.01 -0.62 -0.08  0.32  0.19 -0.37 -0.27  0.09  0.03
(16,1)  0.25 -0.31 -0.33  0.02  0.66  0.08 -0.10  0.31  0.19  0.19
(17,1)  0.30 -0.33 -0.46 -0.08  0.62  0.17 -0.18  0.05  0.04  0.18
(18,1)  0.30 -0.20 -0.22 -0.05  0.81  0.21 -0.09  0.06  0.14  0.11
(19,1)  0.29 -0.17 -0.25  0.01  0.68  0.20  0.04 -0.01  0.29  0.17
(20,1)  0.11 -0.15 -0.95  0.03  0.23  0.08  0.04  0.03  0.01 -0.02
(21,1)  0.46 -0.41 -0.27 -0.02  0.70  0.13 -0.07 -0.06 -0.04  0.11
(22,1)  0.10  0.20 -0.02 -0.01  0.34  0.25 -0.04  0.01  0.68  0.01

Cmd> aaup <- matrix(vecread("aaup"),13)’
Read from file "aaup"

Cmd> aaup <- aaup[,run(8)]

Cmd> laaup <- log(aaup)

Cmd> R <- cor(laaup)

Cmd> out <- facanal(R,5,quiet:T)
WARNING: With m = 5 and p = 8, m > (2*p + 1 - sqrt(8*p+1))/2

Cmd> L <- out$loadings

Cmd> Lr <- rotation(L)

Cmd> print(Lr,format:"f6.3")
Lr:
(1,1)  0.739 -0.091 -0.467  0.251 -0.402
```

```
(2,1)  0.526 -0.134 -0.513  0.265 -0.607
(3,1)  0.475 -0.146 -0.727  0.255 -0.383
(4,1)  0.616 -0.130 -0.492  0.466 -0.379
(5,1)  0.727  0.044 -0.467  0.280 -0.415
(6,1)  0.503  0.041 -0.517  0.310 -0.617
(7,1)  0.443  0.055 -0.748  0.299 -0.390
(8,1)  0.596  0.020 -0.503  0.484 -0.396

Cmd> Lr <- rotation(L,method:"quartimax")

Cmd> Lr <- rotation(L,method:"quartimax")

Cmd> print(Lr,format:"f6.3")
Lr:
(1,1)  0.977 -0.158 -0.016 -0.105  0.077
(2,1)  0.977 -0.062 -0.079  0.010 -0.178
(3,1)  0.962  0.139 -0.206  0.011  0.027
(4,1)  0.982 -0.089  0.011  0.136  0.097
(5,1)  0.980 -0.090  0.104 -0.128  0.063
(6,1)  0.978  0.034  0.074 -0.010 -0.191
(7,1)  0.964  0.264 -0.040 -0.018  0.016
(8,1)  0.983 -0.003  0.132  0.102  0.076

Cmd> Lr <- rotation(L,method:"equimax")

Cmd> print(Lr,format:"f6.3")
Lr:
(1,1)  0.505  0.373 -0.386 -0.584 -0.337
(2,1)  0.380  0.361 -0.469 -0.439 -0.555
(3,1)  0.491  0.391 -0.620 -0.293 -0.352
(4,1)  0.638  0.368 -0.337 -0.397 -0.432
(5,1)  0.474  0.494 -0.327 -0.561 -0.332
(6,1)  0.347  0.518 -0.393 -0.397 -0.547
(7,1)  0.451  0.574 -0.549 -0.237 -0.334
(8,1)  0.593  0.502 -0.287 -0.368 -0.423
```