

1. Do either temperature or rpm affect viscosity? If so, how?

Increasing either temperature or rpm decreases viscosity, although we can only distinguish the low levels from the other two. See more extensive notes below.

2. What model did you use for these data? Defend your choice.

I used a three factor model with sample random. I pooled the temperature/rpm interaction and the three factor interaction into error. Main effects are significant, but no interactions; see more extensive notes below.

3. Two common fish species in cold water streams are slimy sculpin and brown trout. These species tend to inhabit “riffles”, which are shallow running stretches of the stream, sort of like miniature rapids. We are interested in whether the presence of the two species together inhibits or enhances total fish growth (combined across species). To study this, we place small cages called enclosures in riffles. Into each cage we can place either equal weights  $x$  of slimy sculpin and brown trout, or a weight  $2x$  of brown trout, or a weight  $2x$  of slimy sculpin. After a month, we weigh the fish in each cage to assess total growth.

In our experiment there are five riffles. In each riffle we place three enclosures. The three treatments are randomized to the enclosures subject to the restriction that each treatment occurs once in each riffle. Describe the design that was used and give a skeleton ANOVA (sources and df only).

This is a randomized complete block design. Riffles are blocks, and the three species mixtures are the treatments. Cages are units.	Source	DF
	Block	4
	Species	2
	Error	8

4. We are studying the Kraft pulping process for making paper. In this experiment, we look at the charge level (705, 853, or 1000), and which additive is used (control, DQ2016 at .1, DQ2016 at .2, AQ at .1, or DTPA at .2). We can make 10 batches of pulp per day and do the experiment over three days, producing two batches of pulp for each of the 15 combinations of charge level and additive. The fifteen factor/level combinations are randomly assigned to the 30 batches subject to the restriction that each combination is used twice. Describe the design that was used and give a skeleton ANOVA (sources and df only).

This is a completely randomized design. It might have made sense to block on days, but no such blocking was done. Batches are units, and the 15 factor/level combinations are treatments.	Source	DF
	Charge	2
	Additive	4
	Charge.Additive	8
Error	15	

5. We wish to study “sensory specific satiety.” This is the phenomenon wherein if you eat a lot of some food, then that food and similar foods become less liked. In our case we are investigating four kinds of potato chips: classic, sour cream, barbeque, and cheese. Each subject will participate in several sessions. At each session a subject will eat a load food (one of the four kinds of chips). After eating the load food, the subject will rate his or her liking of each of the four kinds of chips. We anticipate large subject to subject differences. We also anticipate that ratings could differ from session to session (for example, we suspect that first session ratings could be higher than last session ratings). Each subject will be available for two sessions, and we have 24 subjects. Choose an appropriate design for this experiment.

Load food is treatment. We need to block on two things: subject and session. Because we can only do two load foods per subject, we’ll need some kind of incomplete block design to do that blocking. We also want

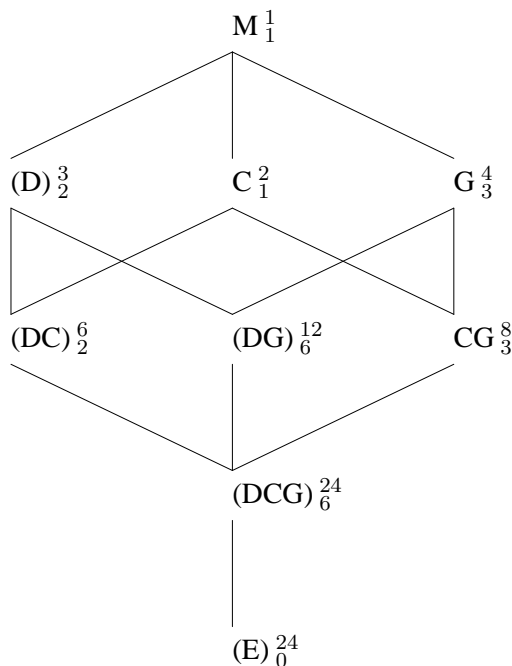
to block on session by having each load food appear an equal number of times during each session.

There are four load foods, so there are six pairs of load foods. You could use each pair in either order, so that is twelve ordered pairs. Use each order pair twice. By construction, each load food occurs six times in each session. Also, every pair of treatments is used for an equal number of subjects, so we have a BIBD for subject blocking. This is a row orthogonal design (or like a Youden square where we've lopped two rows off the Latin square).

6. Poly-3-hydroxybutyrate (PHB) is a biologically produced polymer that is becoming popular because it is biodegradable. This experiment studies the lab method used for measuring the concentration of PHB in a sample. The overall method is to digest samples of known concentration and then measure the concentration via gas chromatography. The procedure involves internal standards, recalibration of instruments, various independent dilutions, and so on. In particular, at this stage we have to deal with the possibility that everything could interact with everything else.

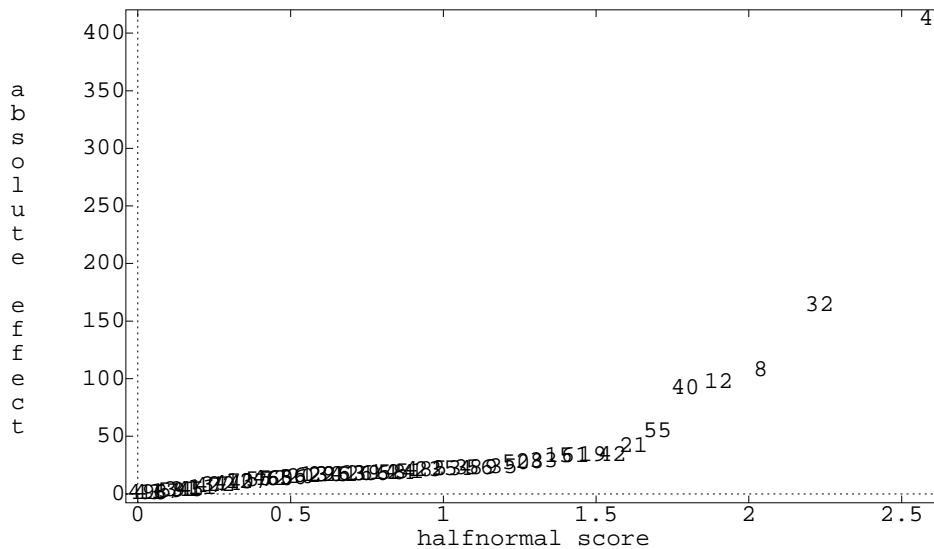
We will do this on three randomly chosen days. On each day we will make up eight samples of PHB. Each sample is randomly assigned to one of the combinations of four concentrations and two digestion methods. Once we have made the sample, we will measure the concentration twice on the GC.

Draw the Hasse diagram for this experiment (including superscripts and subscripts).



7. We have a computer simulation model of traffic flow that we wish to use to test how the algorithm controlling freeway ramp meters should be set. The algorithm works by producing a “random” traffic pattern with different arrival times, different styles of driving, different destinations, etc. We combine this traffic with the simulator and the metering algorithm to measure system total travel time (how long it takes for all our drivers to get through the stretch of highway we are simulating). In this experiment we vary six factors, each at two levels. (Factors are things like metering initiation threshold, maximum wait time, lane capacity, etc; we’ll just call them A through F.)

A single replication of the  $2^6$  design was run, and here is a halfnormal plot of the results in standard order. Which factors and/or interactions are significant?



It is pretty clear from the graph that effects 4, 32, 8, 12, and 40 are outliers, and thus are significant. These are the main effects of C, F, D and the CD and DF interactions.

### Cheese Viscosity Data

**Summary** Analyzing on the logarithmic scale, the main effects of sample, temperature, and rpm are all significant; none of the interactions is significant. Please note, this means that the model is multiplicative on the original scale. Viscosity decreases with increasing temperature and rpm, but we can only distinguish the lowest levels of temperature and rpm from the other two.

**Analysis** There are three factors: sample (random, two levels), temperature (fixed, three levels), and rpm (fixed, three levels). The important issue is how to handle sample. The randomization is consistent with sample being used as a block, but that is also consistent with units nested in samples and crossed with the other factors. The deciding factor is that the scientific questions are specifically concerned with how the treatment effects may differ across samples; this means that we are specifically interested in interaction between sample and the other factors. Thus the standard model of additive blocks will not be adequate to answer these questions, and we will use a three factor model with sample random and all factors crossed.

An initial model including all of these factors in unsatisfactory, because we have no estimate of error (which is the denominator for sample and all interactions involving sample). Thus we remove the three-factor interaction from the model and try again, getting:

	DF	MS	Error DF	Error MS	F	P value
CONSTANT	1	2.074e+07	1	1.476e+05	140.5	0.05358
sample	1	1.476e+05	4	2843	51.92	0.001967
temp	2	2.363e+05	2	1.354e+04	17.46	0.05417
sample.temp	2	1.354e+04	4	2843	4.761	0.0875
rpm	2	1.471e+05	2	1.001e+04	14.69	0.06372
sample.rpm	2	1.001e+04	4	2843	3.521	0.1312
temp.rpm	4	2672	4	2843	0.9398	0.5233
ERROR1	4	2843	0	0	MISSING	MISSING

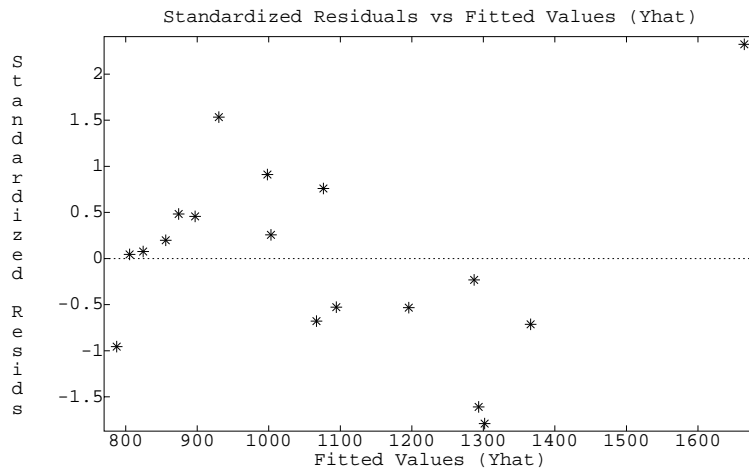
We only have 4 degrees of freedom for error (which is the denominator for several terms), and the temperature by rpm interaction (immediately above error in the Hasse diagram) has an F less than 1, so we pool it

into error and try again, getting:

	DF	MS	Error DF	Error MS	F	P value
CONSTANT	1	2.074e+07	1	1.476e+05	140.5	0.05358
sample	1	1.476e+05	8	2758	53.53	8.258e-05
temp	2	2.363e+05	2	1.354e+04	17.46	0.05417
sample.temp	2	1.354e+04	8	2758	4.909	0.04064
rpm	2	1.471e+05	2	1.001e+04	14.69	0.06372
sample.rpm	2	1.001e+04	8	2758	3.63	0.07554
ERROR1	8	2758	0	0	MISSING	MISSING

Considering the F statistics, we are done with pooling.

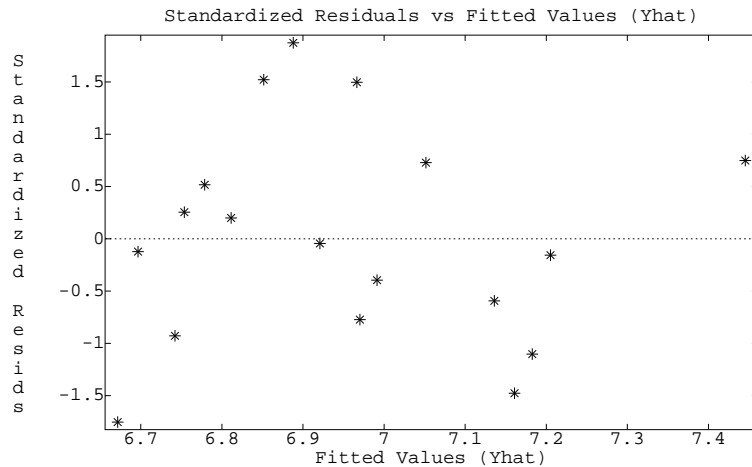
Before we get too far into interpretation, we should look at our residuals, and they do not look good:



We either have an outlier on the right, or we are analyzing on the wrong scale. Both Tukey 1 degree of freedom and Box-Cox suggest a log transformation. On that scale we get the ANOVA

	DF	MS	Error DF	Error MS	F	P value
CONSTANT	1	871.2	1	0.113	7711	0.00725
sample	1	0.113	8	0.001559	72.47	2.784e-05
temp	2	0.1895	2	0.00392	48.35	0.02026
sample.temp	2	0.00392	8	0.001559	2.514	0.1421
rpm	2	0.1179	2	0.003236	36.43	0.02672
sample.rpm	2	0.003236	8	0.001559	2.076	0.1879
ERROR1	8	0.001559	0	0	MISSING	MISSING

The residuals are improved, although certainly not perfect:



On the log scale, sample, temperature, and rpm seem to have effects. The F-statistics for temperature and rpm are very large, even though the p-values are only moderately small; this is because there are only two degrees of freedom for error for these terms. The interactions of sample with temperature and rpm are not significant, but their F-statistics are just a bit too big to justify pooling them into error (although if we did pool, temperature and rpm would then be highly significant).

Using their interactions with sample as error, we get the following HSD pairwise differences for temperature

	3	-0.147
	2	-0.0509
	1	0.198

and rpm

	3	-0.103
	2	-0.0562
	1	0.16

We see that viscosity decreases with temperature (as expected), but we can only distinguish the first temperature from the other two, and viscosity also decreases with rpm, but again, we can only distinguish the first rpm from the other two. (If we cheat a bit and pool the interactions of sample into error, then all the temperatures can be distinguished, but we still can only distinguish the first rpm from the other two.)