Two-Series Factorials

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October 21, 2013

If we have a factorial where all factors have two levels, we call it a two-series factorial.

With k factors, we use the shorthand of a 2^k design.

A three-series design is one where all factors have three levels; it is abbreviated as a 3^k design. An abbreviation for a five-series design is left as an exercise for the reader.

A 2^k design is the smallest k-factor design, making them very popular, especially when you have many factors.

Over the years, a lot of specialty notation and methodology have been developed for two-series designs.

For example, the two levels of a factor are generally called low and high, but they could be off and on, or minus and plus, or 0 and 1. All of those have been used at one time or other.

We have k factors, and for concreteness we illustrate with three factors that we label A, B, and C. With three factors there are 8 treatments.

One method for labeling treatments uses a 0 if a factor is low and a 1 if a factor is high. Thus 011 is A low, B high, C high.

A second method for labeling treatments uses the lower case factor letter if the factor is high and omits the letter if the factor is low. Thus bc is a low, b high, c high. This method uses (1) to denote the treatment where all factors are low (equivalent to 000).

Note: we previously used the lower case letters to denote the number of levels, but in a 2^k that's always two, so we can use them for something else.

Standard order begins with all factors low and then enumerates with A varying fastest, then B, then C.

000	(1)
100	a
010	b
110	ab
001	c
101	ac
011	bc
111	abc

Note that A follows the pattern low, high, low, high. B follows the pattern low, low, high, high. C follows the pattern of four lows and then four highs. D would be eight lows then eight highs, and so on.

Each effect (main effect or interaction) in a two-series design has a single degree of freedom. Thus each effect is determined by a single contrast.

We can represent these contrasts as a table of plus and minus signs (plus ones and minus ones, if you will).

If we know the signs for the factors, we get the signs for an interaction effect by element-wise multiplication of the signs in the factors in the interaction.

Trt	A	B	C	...	ABC
(1)	−	−	−		−
a	+	−	−		+
b	−	+	−		+
ab	+	+	−		−
c	−	−	+		+
ac	+	−	+		−
bc	−	+	+		−
abc	+	+	+		+

All of the effect contrasts are orthogonal, and thus independent for normal data.

Under the null hypothesis of no effects at all, the contrast values (or effect estimates) are independent, with mean 0 and constant variance.

Significant effects are outliers in a normal probability plot (I prefer a half-normal plot). This approach assumes that most effects are null and only a few are active (effect sparsity) and is viable when we have no estimate of pure error (single replication).

Outliers are somewhat difficult to define.[1]

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[1]Supreme Court justice Potter Stewart famously said hard-core pornography was hard to define, but "I know it when I see it."

Lenth's PSE

Lenth's pseudo-standard error is an attempt to go beyond "I know it when I see it" in identifying the outliers.

PSE assumes most effects are null and uses the bulk of the smallish effects to estimate error.

1. Temporarily set aside all effects larger (in absolute value) than 3.75 times the median of the absolute value of the effects.
2. Let PSE be 1.5 times the median of the remaining absolute effects.
3. Use the PSE as the denominator for a t-test on effects with $(2^k - 1)/3$ df.

If your data in a two-series are all 0 except for a single non-zero value, then the factorial effects are all <u>equal</u> in absolute value.

Flat spots in the (half) normal plot of effects can indicate an outlier or a large one-cell interaction.