Normal Mean OC and design

Data $X_1 \ldots X_n \sim N(\mu, \sigma^2)$. Test at significance level $\alpha$.

$H_0 \ \mu = \mu_0 \ \text{vs} \ H_a \ \mu > \mu_0$

Test statistic is

$$U = \frac{\sqrt{n}(X-\mu_0)}{\sigma} \sim N\left(\frac{\sqrt{n}(\mu-\mu_0)}{\sigma}, 1\right)$$

Under $H_0$, this is $N(0,1)$, and we reject if $U > z_{1-\alpha}$

OC is $\Pr[\text{Accept } H_0 | \mu] = \Pr[ U < z_{1-\alpha}] = \Phi\left[z_{1-\alpha} \cdot \frac{\sqrt{n}(\mu-\mu_0)}{\sigma}\right]$
Designing to get power $\beta$

$$1 - \beta = \Phi \left[ z_{1-\alpha} - \frac{\sqrt{n}(\mu - \mu_0)}{\sigma} \right]$$

$$\Phi^{-1}[1 - \beta] = \Phi \left[ z_{1-\alpha} - \frac{\sqrt{n}(\mu - \mu_0)}{\sigma} \right]$$

$$-z_\beta = \left[ z_{1-\alpha} - \frac{\sqrt{n}(\mu - \mu_0)}{\sigma} \right]$$

$$\frac{\sqrt{n}(\mu - \mu_0)}{\sigma} = z_{1-\alpha} + z_\beta$$
Two-sided test

Reject if $|U| > z_{1-\alpha/2}$ OC is

\[
\text{Pr}[-z_{1-\alpha/2} < U < z_{1-\alpha/2}] = \\
\Phi\left(z_{1-\alpha/2} - \frac{\sqrt{n}(\mu - \mu_0)}{\sigma}\right) - \Phi\left(-z_{1-\alpha/2} - \frac{\sqrt{n}(\mu - \mu_0)}{\sigma}\right)
\]

You can do approximate but realistic design calcs by ignoring “wrong-side” tail.

Solve $\frac{\sqrt{n}|\mu - \mu_0|}{\sigma} = z_{1-\alpha/2} + z_\beta$