Stat 5102 (Geyer) Spring 2013
Homework Assignment 6
Due Wednesday, March 13, 2013

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

6-1. Suppose X_1, X_2, $\ldots$, X_n are IID $\text{Exp}(\lambda)$. Find the likelihood function and log likelihood function.

6-2. Suppose X_1, X_2, $\ldots$, X_n are IID $\text{Geo}(p)$. Find the likelihood function and log likelihood function.

6-3. Suppose X_1, X_2, $\ldots$, X_n are IID $\text{Cauchy}(\mu, \sigma)$, where both μ and σ are unknown parameters. Find the likelihood function and log likelihood function.

6-4. Suppose X_1, X_2, $\ldots$, X_n are IID $\text{Laplace}(\mu, \sigma)$, where both μ and σ are unknown parameters. Find the likelihood function and log likelihood function.

6-5. Suppose X_1, $\ldots$, X_n are IID $\text{Exp}(\theta)$ and Y_1, $\ldots$, Y_n are IID $\text{Exp}(1/\theta)$ and the X's and Y's are independent of each other. Find the likelihood function and log likelihood function.

6-6. Suppose X_1, X_2, $\ldots$, X_n are IID with PDF

$$f_\theta(x) = (\theta - 1)x^{-\theta}, \qquad 1 < x < \infty,$$

where $\theta > 1$ is an unknown parameter. Find the likelihood function and log likelihood function.

6-7. Find the first and second derivatives of the log likelihood function

(a) for problem 6-1.

(b) for problem 6-2.

(c) for problem 6-3, considering μ the unknown parameter and σ known.

(d) for problem 6-5.

(e) for problem 6-6.

6-8. Find the Fisher information

(a) for problem 6-1.

(b) for problem 6-2.

(c) for problem 6-5.

(d) for problem 6-6.

6-9. For the setup of Problem 6-1

(a) Find the MLE.

(b) Show that your MLE is at least a local maximizer of the log likelihood.

(c) If the log likelihood is strictly concave, show that it is.

(d) Find the asymptotic distribution of your MLE.

(e) Show that your MLE is the same as a method of moments estimator we derived earlier, and the asymptotic distribution obtained in part (d) is the same as the asymptotic distribution derived earlier via the delta method.

(f) Give an asymptotic 95% confidence interval for the parameter based on the asymptotic distribution of the MLE.

6-10. For the setup of Problem 6-2

(a) Find the MLE.

(b) Show that your MLE is at least a local maximizer of the log likelihood.

(c) If the log likelihood is strictly concave, show that it is.

(d) Find the asymptotic distribution of your MLE.

(e) Show that your MLE is the same as a method of moments estimator we derived earlier, and the asymptotic distribution obtained in part (d) is the same as the asymptotic distribution derived earlier via the delta method.

(f) Give an asymptotic 95% confidence interval for the parameter based on the asymptotic distribution of the MLE.

6-11. For the setup of Problem 6-5

(a) Find the MLE.

(b) Show that your MLE is at least a local maximizer of the log likelihood.

(c) If the log likelihood is strictly concave, show that it is.

(d) Find the asymptotic distribution of your MLE.

(e) Give an asymptotic 95% confidence interval for the parameter based on the asymptotic distribution of the MLE.

6-12. For the setup of Problem 6-6

(a) Find the MLE.

(b) Show that your MLE is at least a local maximizer of the log likelihood.

(c) If the log likelihood is strictly concave, show that it is.

(d) Find the asymptotic distribution of your MLE.

(e) Give an asymptotic 95% confidence interval for the parameter based on the asymptotic distribution of the MLE.

6-13. For the setup of Problem 6-4, suppose that μ is the unknown parameter and σ is known. Show that the sample median is an MLE. Note that the log likelihood is not everywhere differentiable, so this problem cannot be done by setting the first derivative equal to zero and solving. You need a different argument. **Hint:** Consider properties of the sample median.

6-14. Suppose X is $\text{Poi}(\mu)$. Find the MLE. Show that it is the unique global maximizer of the likelihood. **Caution:** In this problem the data are X. There is no X_1, X_2, $\ldots$.

6-15. Suppose X is $\text{Poi}(\mu)$. Suppose we are interested in the parameter $p = e^{-\mu}$, which is $\Pr(X = 0)$.

(a) Find the MLE for p.

(b) Find the Fisher information for p.

(c) Find the asymptotic distribution of your MLE.

(d) Give an asymptotic 95% confidence interval for the parameter based on the asymptotic distribution of the MLE.

5-9. (Moved from last week and fixed.) Suppose the following data

control	treatment
6.48	10.24
2.06	3.07
2.96	4.04
2.84	2.85
4.47	5.33
1.96	3.29
10.28	9.03
2.45	3.85
1.72	3.30
1.23	2.72
6.14	7.74
5.91	7.36
7.66	4.06
4.46	7.80
4.95	8.46

are from a paired comparison experiment (each row of the table is data for one individual under two conditions). If you want to use R on this problem, the data are in the file

`http://www.stat.umn.edu/geyer/5102/data/prob5-9.txt`

Compute a nonparametric confidence interval for the median difference. Note that this is not the same as the difference of medians (although, as noted in problem 5-2, if we assume both populations have means and the distribution of differences is symmetric, then the median difference is equal to the mean difference, which is the difference of means). If we call the control data X_i and the treatment data Y_i and we define $Z_i = Y_i - X_i$, then we want a confidence interval for the median of the Z_i, which is not the same as the difference of the medians of the X_i and Y_i. Also note that the names of the variables in the data file are `control` and `treatment` rather than `x` and `y`.

(a) First assume only that the distribution of the differences is continuous and calculate a confidence interval associated with the sign test.

(b) Then assume only that the distribution of the differences is continuous and symmetric, in which case the median is also the center of symmetry, and calculate a confidence interval associated with the signed rank test.

In each case calculate a conventional two-sided interval that has confidence level at least 95% but no larger than 95% than it has to be. In each case state the exact confidence level that is achieved.