Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

12-1. The data set in the URL http://www.stat.umn.edu/geyer/5102/data/prob12-1.txt has two variables x (the predictor variable) and y (the response variable). The response is zero-or-one-valued, so these data are suitable for logistic regression.

(a) Fit a GLM with logit link having natural parameter (linear predictor) that is quadratic in x, that is

\[ \theta_i = \beta_1 + \beta_2 x_i + \beta_3 x_i^2 \]

Report the regression coefficients table for this fit.

(b) Give a 95% confidence interval for the natural parameter value for an individual with \( x_i = 30 \).

(c) Give a 95% confidence interval for the mean parameter value for an individual with \( x_i = 30 \) that is of the form point estimate \( \pm \) critical value \( \times \) standard error.

(d) Give a 95% confidence interval for the mean parameter value for an individual with \( x_i = 30 \) that is made by transforming the endpoints of the interval from part (b) from the natural parameter scale to the mean value parameter scale.

12-2. Redo problem 12-1 substituting probit link for logit link.

12-3. Redo problem 12-1 substituting cauchit link for logit link.

12-4. The data set in the URL http://www.stat.umn.edu/geyer/5102/data/prob12-4.txt has three variables x1 and x2 (the predictor variables) and y (the response variable). The response is zero-or-one-valued, so these data are suitable for logistic regression.
(a) Fit a GLM with logit link having natural parameter (linear predictor) that is linear in \( x_1 \) and \( x_2 \), that is

\[ \theta_i = \beta_1 + \beta_2 x_{1i} + \beta_3 x_{2i} \]

Report the regression coefficients table for this fit.

(b) Same as part (a) but with natural parameter quadratic in \( x_1 \) and \( x_2 \) (there will be six regression coefficients).

(c) Same as part (a) but with natural parameter cubic in \( x_1 \) and \( x_2 \) (there will be ten regression coefficients).

(d) Compare the three models fit in parts (a) through (c) using likelihood ratio tests. There will be two tests, one comparing (a) to (b) and one comparing (b) to (c).

(e) Interpret these tests. What do they say about the correctness of each of the models?

12-5. The data set in the URL

http://www.stat.umn.edu/geyer/5102/data/prob12-5.txt

has twenty-one variables \( x_1 \) through \( x_{20} \) (the predictor variables) and \( y \) (the response variable). The response is normal, so these data are suitable for ordinary least squares regression.

(a) Use the R function \texttt{regsubsets} in the R package of the same name to examine all \( 2^{20} \) (more than a million) models each containing a subset of the predictor variables plus an intercept.

(b) Which model fits best according to AIC?

(c) Which model fits best according to BIC?