

Stat 5102 (Geyer) Spring 2012
Homework Assignment 9
Due Wednesday, April 11, 2012

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

9-1. Suppose X is $\text{Bin}(n, p)$ and the prior distribution for p is flat (a proper prior, since p is bounded).

- (a) Find the posterior distribution for p .
- (b) Find the mean of the posterior distribution for p .
- (c) Find the standard deviation of the posterior distribution for p .
- (d) Find the mode of the posterior distribution for p .
- (e) In the case $x = 0$, $n = 10$, find the posterior median for p .

9-2. Suppose X_1, \dots, X_n are IID $\text{Exp}(\lambda)$ and the prior distribution for λ is flat (an improper prior). The posterior distribution for λ was found in problem 8-10 (a).

- (a) Find the mean of the posterior distribution for λ .
- (b) Find the standard deviation of the posterior distribution for λ .
- (c) Find the mode of the posterior distribution for λ .
- (d) In the case $\bar{x}_n = 23.7$, $n = 10$, find the posterior median for λ .

9-3. Find the Jeffreys prior for the $\text{NegBin}(r, p)$ distribution, considering r fixed and known and p the unknown parameter. It is proper or improper?

9-4. Find the Jeffreys prior for the $\text{Gam}(\alpha, \lambda)$ distribution, where α is known and λ unknown, so we want a prior distribution for λ . It is proper or improper?

9-5. Find the posterior mean and variance of μ when the data are IID normal and the prior is a general normal-gamma prior. Say for which values of the hyperparameters of the prior the posterior mean and variance of μ exist.

9-6. Suppose X_1, \dots, X_n are IID $\mathcal{N}(\mu, 4)$, the prior distribution for μ is $\mathcal{N}(10, 9)$, and the sample mean of a sample of size 10 is $\bar{X}_n = 12$. Calculate a 90% HPD region for μ (note not 95%).

9-7. Suppose X_1, \dots, X_n are IID $\mathcal{N}(\mu, \lambda^{-1})$, the prior distribution for (μ, λ) is the conjugate normal-gamma prior with

$$\begin{aligned}\lambda &\sim \text{Gam}(3, 3) \\ \mu \mid \lambda &\sim \mathcal{N}(10, 16\lambda^{-1})\end{aligned}$$

the sample mean of a sample of size 15 is $\bar{X}_n = 12$ and the sample variance is $S_n^2 = 50$ (note not V_n).

- (a) Calculate a 95% HPD region for μ .
- (b) Calculate the exact frequentist 95% confidence interval for μ .

9-8. Suppose X_1, \dots, X_n are IID $\text{Exp}(\lambda)$ and the prior distribution for λ is $\text{Gam}(3, 3)$.

- (a) Calculate the posterior probabilities of the events

$$\begin{aligned}H_0: \lambda &\geq 1 \\ H_1: \lambda &< 1\end{aligned}$$

when $n = 4$ and $\bar{x}_n = 1.9$.

- (b) Calculate the prior probabilities of the same events.
- (c) Calculate the Bayes factor

$$\frac{\Pr(H_0 \mid \mathbf{x})}{\Pr(H_1 \mid \mathbf{x})} \cdot \frac{\Pr(H_1)}{\Pr(H_0)}$$

- (d) Calculate an exact frequentist P -value for these hypotheses based on the exact sampling distribution of $X_1 + \dots + X_n$.

9-9. Suppose X_1, \dots, X_n are IID $\text{Exp}(\lambda)$. In this problem we are interested in the hypotheses (models)

$$\begin{aligned}m_1 &= H_0: \lambda = 1 \\ m_2 &= H_1: \lambda \neq 1\end{aligned}$$

Suppose the prior distribution for λ given model m_2 is $\text{Gam}(3, 3)$. The prior distribution for λ given model m_1 is concentrated at the point $\lambda = 1$. Suppose $n = 4$ and $\bar{x}_n = 1.9$. Calculate the Bayes factor (model 1 over model 2). Hint: For both models, proceed as if the data were $X_1 + \dots + X_n$.