Stat 5102 (Geyer) Spring 2012
Homework Assignment 8
Due Wednesday, March 28, 2012

Solve each problem. Explain your reasoning. No credit for answers with
no explanation. If the problem is a proof, then you need words as well as
formulas. Explain why your formulas follow one from another.

8-1. Show that each of the following is an exponential family. Identify the
natural parameter and natural statistic.
(a) The Poi($\mu$) family of distributions.
(b) The Exp($\lambda$) family of distributions.
(c) The Gam($\alpha, \lambda$) family of distributions with both parameters unknown.
   The natural parameter vector and natural statistic vector are both two-
dimensional.

8-2. Suppose $X$ is Poi($\mu$) and the prior distribution for $\mu$ is Gam($\alpha, \lambda$),
where $\alpha$ and $\lambda$ are hyperparameters. Find the posterior distribution for $\mu$.

8-3. Suppose $X_1, \ldots, X_n$ are IID Gam($\alpha, \lambda$), where $\alpha$ is known and $\lambda$ is
unknown. Suppose the prior distribution for $\lambda$ is Gam($\alpha_0, \lambda_0$), where $\alpha_0$
and $\lambda_0$ are hyperparameters. Find the posterior distribution for $\lambda$.

8-4. Suppose $X_1, \ldots, X_n$ are IID Unif(0,$\theta$) and the prior distribution for
$\theta$ is Unif($a,b$), where $a$ and $b$ are hyperparameters. Find the PDF of the
posterior distribution for $\theta$. Under what conditions on $x_1, \ldots, x_n, a,$ and $b$
does the solution make no sense?

8-5. Suppose the distribution for data $X$ is Geo($p$). Show that the beta
family of distributions is conjugate.

8-6. Suppose $X_1, \ldots, X_n$ are IID $N(\mu, 1/\lambda)$, where $\mu$ is known and $\lambda$
is unknown. Find a brand-name family of distributions that is conjugate.

8-7. Suppose $X$ is Geo($p$) and the prior distribution for $p$ is Beta($\alpha_1, \alpha_2$),
where $\alpha_1$ and $\alpha_2$ are hyperparameters. Find the posterior distribution for $p$.

8-8. Suppose $X_1, \ldots, X_n$ are IID $N(\mu, 1/\lambda)$, where $\mu$ is known and $\lambda$
is unknown. Suppose the prior distribution for $\lambda$ is a distribution in the
brand-name conjugate family of distributions found in problem 8-6. Find
the posterior distribution for $\lambda$. 

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8-9. Suppose the situation is the same as in problem 8-8. Find the posterior distribution for \( \sigma = \sqrt{1/\lambda} \). **Hint:** change-of-variable formula.

8-10. Suppose \( X_1, \ldots, X_n \) are IID Exp(\( \lambda \)).

(a) Suppose the prior distribution for \( \lambda \) is flat (an improper prior). Find the posterior distribution for \( \lambda \).

(b) Suppose the prior distribution for \( \lambda \) is proportional to \( \lambda^{-1} \) (an improper prior). Find the posterior distribution for \( \lambda \).

**Review Problems from Last Year’s Tests**

8-11. Suppose \( X_1, \ldots, X_n \) are IID Exp(\( \lambda \)), and suppose the prior distribution for \( \lambda \) is Gam(\( \alpha_0, \lambda_0 \)), where \( \alpha_0 \) and \( \lambda_0 \) are hyperparameters. Find the posterior distribution for \( \lambda \).

8-12. Suppose \( X \) is Poi(\( \mu \)). We have only one observation. And suppose the prior distribution for \( \mu \) is proportional to \( \mu^{-1/2} \), an improper prior.

(a) Find the posterior distribution for \( \mu \).

(b) For what values of the data \( x \) does your answer to part (a) make sense?