$$(X_i - \overline{X}_n)^k$$

$$= \left[(X_i - \mu) - (\overline{X}_n - \mu) \right]^k$$

$$= \sum_{j=0}^{k} \binom{k}{j} (\overline{X}_n - \mu)^j (X_i - \mu)^{k-j} (-1)^j$$

$$n^{1/2 - j/2} n^{j/2} (\bar{X}_n - \mu)^j \xrightarrow{\mathcal{D}} U^j \cdot 0$$

$$= 0$$

$$j > 1$$

$$\operatorname{var}\begin{pmatrix} X_i - \mu \\ (X_i - \mu)^k \end{pmatrix} = \begin{pmatrix} \mu_2 & \mu_{k+1} \\ \mu_{k+1} & \mu_{2k} - \mu_k^2 \end{pmatrix}$$

$$\operatorname{cov}\left(X_i - \mu, (X_i - \mu)^k\right)$$

$$= E\left\{(X_i - \mu)^{k+1}\right\} - \underbrace{E(X_i - \mu)}_{= 0} E\left\{(X_i - \mu)^k\right\}$$

$$\mu_{2k} - \mu_k^2 - 2k\mu_{k-1}\mu_{k+1} + k^2\mu_{k-1}^2\mu_2$$

$k = 2$

$$\mu_4 - \mu_2^2$$