$$\text{cov}\left(a^T\hat{\beta}, a^T\tilde{\beta}\right)$$

$A = B$ oops!

$$= \text{cov}\left(a^T(M^TM)^{-1}M^TY,\ a^TAY\right)$$

$$= E\left\{a^T(M^TM)^{-1}M^T(Y^{(M)}Y_{(M)}^T)A^Ta\right\}$$

$$= a^T(M^TM)M^T\,\text{var}(Y)\,A^Ta$$

$$= \sigma^2(M^TM)M^TA^Ta = 0$$

because $AM = 0$, $M^TA^T = 0$