8-1. Show that each of the following is an exponential family. Identify the natural parameter and natural statistic.

(a) The Poi(μ) family of distributions.

(b) The Exp(λ) family of distributions.

(c) The Gam(α, λ) family of distributions with both parameters unknown. The natural parameter vector and natural statistic vector are both two-dimensional.

8-2. Suppose X is Poi(μ) and the prior distribution for μ is Gam(α, λ), where α and λ are hyperparameters. Find the posterior distribution for μ.

8-3. Suppose X₁, ..., Xₙ are IID Gam(α, λ), where α is known and λ is unknown. Suppose the prior distribution for λ is Gam(α₀, λ₀), where α₀ and λ₀ are hyperparameters. Find the posterior distribution for λ.

8-4. Suppose X₁, ..., Xₙ are IID Unif(0, θ) and the prior distribution for θ is Unif(a, b), where a and b are hyperparameters. Find the PDF of the posterior distribution for θ. Under what conditions on x₁, ..., xₙ, a, and b does the solution make no sense?

8-5. Suppose the distribution for data X is Geo(p). Show that the beta family of distributions is conjugate.

8-6. Suppose X₁, ..., Xₙ are IID N(μ, 1/λ), where μ is known and λ is unknown. Find a brand-name family of distributions that is conjugate.

8-7. Suppose X is Geo(p) and the prior distribution for p is Beta(α₁, α₂), where α₁ and α₂ are hyperparameters. Find the posterior distribution for p.

8-8. Suppose X₁, ..., Xₙ are IID N(μ, 1/λ), where μ is known and λ is unknown. Suppose the prior distribution for λ is a distribution in the brand-name conjugate family of distributions found in problem 8-6. Find the posterior distribution for λ.
8-9. Suppose the situation is the same as in problem 8-8. Find the posterior distribution for $\sigma = \sqrt{1/\lambda}$. \textbf{Hint:} change-of-variable formula.

8-10. Suppose $X_1, \ldots, X_n$ are IID Exp($\lambda$).

(a) Suppose the prior distribution for $\lambda$ is flat (an improper prior). Find the posterior distribution for $\lambda$.

(b) Suppose the prior distribution for $\lambda$ is proportional to $\lambda^{-1}$ (an improper prior). Find the posterior distribution for $\lambda$. 