

Name \_\_\_\_\_ Student ID \_\_\_\_\_

The exam is closed book and closed notes. You may use one  $8\frac{1}{2} \times 11$  sheet of paper with formulas, etc. You may use a calculator, but shouldn't need to. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

1. [15 pts.] The function

$$f(x, y) = \frac{2}{2 - \log 2} \cdot \frac{x + y}{1 + y} \quad 0 < x < 1, \quad 0 < y < 1$$

is the joint probability density function of random variables  $X$  and  $Y$ . Find the conditional probability density function  $g_1(x | y)$  of  $X$  given  $Y$ .

2. [15 pts.] The function

$$f(x) = \frac{2\sqrt{3}}{\pi(1 + x^2 + x^4)}, \quad 0 < x < \infty$$

is the probability density function of a random variable  $X$ . Find the probability density function of the random variable  $Y = X^{1/3}$ .

3. [20 pts.] A random variable  $X$  has probability density function  $f$  defined by

$$f(x) = 12(x^2 - x^3), \quad 0 < x < 1.$$

- (a) Calculate  $E(X)$ .  
(b) Calculate  $\text{var}(X)$ .

4. [20 pts.] Random variables  $X$  and  $Y$  have moments

$$\begin{aligned} E(X) &= 6 \\ E(Y) &= 5 \\ \text{var}(X) &= 4 \\ \text{var}(Y) &= 3 \\ \text{cov}(X, Y) &= 2 \end{aligned}$$

- (a) Calculate  $E(7X + 8)$ .
- (b) Calculate  $\text{var}(7X + 8)$ .
- (c) Calculate  $\text{sd}(7X + 8)$ .
- (d) Calculate  $\text{var}(X + Y)$ .
- (e) Calculate  $\text{var}(X - Y)$ .

5. [15 pts.] A random variable  $X$  has probability density function  $f$  defined by

$$f(x) = 3x^{-4}, \quad 1 < x < \infty.$$

Find the median of  $X$ .

6. [15 pts.] There exists a constant  $c$  such that the function

$$f(x) = \frac{cx^2}{(1 + x^2 + x^4 + x^{12})}, \quad 0 < x < \infty$$

is a probability density function. For what positive integers  $k$  does  $E(X^k)$  exist?