

Stat 3011 Midterm 2

Problem 1

This problem requires a probability calculation for a general normal distribution (Section 6.3 in the textbook). It is a “forward” problem (given x find a probability). The solution has two steps.

- Standardize
- Forward Look-up.

The standardization step is

$$z = \frac{x - \mu}{\sigma} = \frac{70 - 63.5}{2.5} = 2.6$$

Looking up $P(z > 2.6)$ in the normal table gives 0.0047.

Problem 2

This problem is about the sampling distribution of a sample proportion (Section 7.3 in the textbook). The distribution of p is approximately normal with mean and standard deviation

$$\begin{aligned}\mu_p &= \pi \\ \sigma_p &= \sqrt{\frac{\pi(1 - \pi)}{n}}\end{aligned}$$

where π is the population proportion and n is the sample size.

In this problem $\pi = 0.52$ and $n = 500$, so

$$\begin{aligned}\mu_p &= 0.52 \\ \sigma_p &= \sqrt{\frac{0.52 \times 0.48}{500}} = 0.02234278\end{aligned}$$

The rest of the problem is a probability calculation for a general normal distribution, just like problem 1. The standardization step is

$$z = \frac{p - \mu_p}{\sigma_p} = \frac{0.50 - 0.52}{0.02234278} = -0.8951436$$

Looking up $P(z < -0.895)$ in the normal table, we see the answer is somewhere between 0.1867 and 0.1841 (either of these or anything in between was accepted for full credit). The exact answer (obtained from Rweb) is 0.1854.

Problem 3

This problem is a small-sample confidence interval for a mean (Section 8.4) in the textbook. The confidence interval is

$$\bar{x} \pm (t \text{ critical value}) \frac{s}{\sqrt{n}}$$

In this problem $\bar{x} = 253$, $s = 123$, and $n = 3$. The t critical value for $n - 1 = 2$ degrees of freedom and 95% confidence is 4.30 (looked up in the table of t critical values). Thus the confidence interval is

$$253 \pm 4.30 \times \frac{123}{\sqrt{3}}$$

which works out to 253 ± 305.36 or $(-52.36, 558.36)$.

Problem 4

This problem is about the sample size needed to get a stated accuracy for a confidence interval for a mean (Section 8.2 in the textbook). If B is the half-width, the sample size is given by

$$n = \left(\frac{1.96\sigma}{B} \right)^2$$

Of course, σ is not known, but we can estimate it by the sample standard deviation of the preliminary sample, $s = 1.44$. This gives

$$n = \left(\frac{1.96 \times 1.44}{0.10} \right)^2 = 796.5942$$

Rounding to the nearest integer gives $n = 797$.

Problem 5

This problem is about setting up the hypotheses for a hypothesis test (Section 9.1 in the textbook). This is a test about proportions. Thus the *parameter* (what Devore and Peck call the *population characteristic*) is π . The hypothesized value (the only number in the problem statement) is 0.47 (probabilities are numbers between zero and one, you must convert percents to this form). Thus the null hypothesis is

$$H_0 : \pi = 0.47$$

The only remaining issue is the choice of $>$, $<$, or \neq in the alternative hypothesis. The way to do this is to find the words in the problem statement that are a paraphrase of the alternative hypothesis. These are

improvement over the standard treatment

Since “improvement” corresponds to probability “greater than,” the correct choice is $>$, and the alternative hypothesis is

$$H_a : \pi > 0.47.$$

Problem 6

This is a question about interpreting P -values (Section 9.3 in the textbook, also Handout B).

(a) The null hypothesis is rejected if $P \leq \alpha$. That is the case here ($0.0315 \leq 0.05$). Thus H_0 is rejected. The null hypothesis corresponds to blind guessing, no ESP. The alternative corresponds to better accuracy than blind guessing, hence ESP. Thus accepting the alternative means accepting ESP.

You might question whether the experiment indicates ESP by raising non-statistical issues. Perhaps there was cheating or inadvertent queing. But the hypothesis test provides evidence for ESP.

(b) Two tails is twice one tail: $P = 0.063$.

(c) Now $P > \alpha$ ($0.063 > 0.05$) so we accept H_0 . And now the experiment provides no evidence for ESP.

Problem 7

This is a large-sample test about means (Section 9.3 in the textbook). The test statistic is

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{46,300 - 50,000}{11,100/\sqrt{40}} = -2.108$$

Looking up $P(z < -2.108)$ in the normal table, we see the answer is somewhere between 0.0179 and 0.0174, much closer to the latter than the former (anything between 0.0174 and 0.0176 was accepted for full credit). The exact answer (obtained from Rweb) is 0.0175.