Stat 3011 Final Exam

Problem 1

The sample mean $\bar{x}$ is approximately normally distributed with mean $\mu = 20$ mm and standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.2}{\sqrt{50}} = 0.3111 \text{ mm}$$

We need to look up $P(x > 21)$ when x is normal with mean 20 and standard deviation 0.3111. To do this we standardize 21 giving $(21 - 20)/0.3111 = 3.214$ and look up $P(z > 3.214)$ when z is standard normal. Table II gives .0007.

Problem 2

This is a problem about proportions. The formula is

$$n = \pi(1 - \pi) \left(\frac{1.96}{B} \right)^2 .$$

No estimate of π is given so we use .5 (the worst case).

$$n = (.5)(.5) \left(\frac{1.96}{.03} \right)^2 = 1067.11$$

Giving an answer of 1068 (round up to be conservative).

Problem 3

With $n = 5$ we should use a t distribution with $n - 1 = 4$ degrees of freedom. From Table III the critical value for a 95% confidence interval is 2.78, which gives a confidence interval

$$\begin{gathered} \bar{x} \pm 2.78 s/\sqrt{n} \\ 9.82 \pm (2.78)(.13)/\sqrt{5} \\ 9.82 \pm 0.1616 \end{gathered}$$

or $(9.658, 9.982)$.

Problem 4

This is a testing problem about proportions. The observed population proportion is $p = .41$, the null and alternative hypotheses are $H_0 : \pi = .30$ and $H_a : \pi > .30$. The test statistic is

$$z = \frac{p - \pi}{\sqrt{\pi(1 - \pi)/n}} = \frac{.41 - .30}{\sqrt{(.30)(.70)/100}} = 2.40040$$

Looking up $P(z > 2.40)$ in Table II gives .0082 for the P-value.

Problem 5

This is a testing problem about means. Since $n = 50$, this is "large sample" and we can use the normal distribution rather than t. The test statistic is

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{26 - 0}{73/\sqrt{50}} = 2.5185$$

Looking up $P(z > 2.52)$ in Table II gives .0059.

Problem 6

This is a testing problem about means. Since $n = 10$, this is "small sample" and we can use the t distribution with $n - 1 = 9$ degrees of freedom to compute P-values. The null and alternative hypotheses are $H_0 : \mu = 15.0$ and $H_a : \mu \neq 15.0$, where μ is the true mean widget length. The test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{14.54 - 15.00}{1.56/\sqrt{10}} = -0.93247$$

Looking up $P(t < -0.9)$ in Table IV (9 d. f.) gives .196. For two tails we double to get a P-value of .392. A more exact two-tailed P-value obtained from interpolation in the table is .375. Nowhere near significance. No reason to believe the true mean is not 15.00.

Problem 7

(a) $H_0 : \mu = 18$ versus $H_a : \mu < 18$.

(b) $H_0 : \mu = 505$ versus $H_a : \mu < 505$.

(c) $H_0 : \pi = .5$ versus $H_a : \pi \neq .5$.

Problem 8

(a) Yes. Reject if $P \le \alpha$. Since $.035 \le .05$, it is.

(b) No. Reject if $P \le \alpha$. Since $.035 \not\le .01$, it isn't.

(c) Two tails are twice one tail. $P = .070$ for the two-tailed test.

(d) No. Reject if $P \le \alpha$. Since $.070 \not\le .05$, it isn't.

Problem 9

(a)

$$b = r\frac{s_y}{s_x} = .60\frac{.021}{.020} = .63$$

and

$$a = \bar{y} - b\bar{x} = .262 - .260 \times .63 = .0982$$

Thus the regression equation is $y = a + bx = .0982 + .63x$.

(b) The prediction is $.0982 + .63 \times .315 = .2967$.

Problem 10

The probability of loosing one bet is $1 - 18/38 = .5263$ (complement rule). The probability of loosing all four is

$$.5263 \times .5263 \times .5263 \times .5263 = .0767$$

(multiplication rule).