

Stat 3011 Midterm 2

Problem 1

(a) Between $P(z < 1.23) = .8907$ and $P(z < 1.24) = .8925$, about .4 of the way from the first to the second, which is .8914.

(b) $P(.56 < z < .78) = P(z < .78) - P(z < .56) = .7823 - .7123 = .0700$.

(c) Since $P(z < -1.03) = .1515$ and $P(z < -1.04) = .1492$, z^* lies about 2/3 of the way from -1.03 to -1.04 , which is -1.037 .

Problem 2

(a)

$$P(x > 1000) = P\left(z > \frac{1000 - 900}{200}\right) = P(z > .5) = .3085$$

(b) We want to find x^* such that $P(x < x^*) = .25$.

This is a backward problem, so first we look up $P(z < z^*) = .25$ and then we “unstandardize” using $x^* = \mu + \sigma z^*$.

Step 1. Since $P(z < -.68) = .2483$ and $P(z < -.67) = .2514$, x^* lies between $-.67$ and $-.68$, about half way between, which is $-.675$.

Step 2. $x^* = 900 - 200 \times .675 = 765$ hours.

Problem 3

This is a question about a sample proportion. The population proportion π of bets won by the gambler is $18/38 = .4737$. The question is about the sample proportion p in a sample of size $n = 1000$: what is $P(p > .5)$?

The mean of p is π and the standard deviation of p is

$$\sqrt{\pi(1 - \pi)/n} = \sqrt{.4737 \times .5263/1000} = .01579$$

so

$$P(p > .5) \approx P\left(z > \frac{.5 - .4737}{.01579}\right) = P(z > 1.666)$$

This is between $P(z > 1.66) = P(z < -1.66) = .0485$ and $P(z > 1.67) = P(z < -1.67) = .0475$, about 2 / 3 of the way between, which is .0478.

Using a computer to do exact rather than approximate calculation the probability that the gambler wins at least 500 bets is .0511 and the probability of winning at least 501 is .0448. The normal approximation is splitting the difference (ignoring the possibility the discreteness of the problem).

Problem 4

This is a problem about means with $n = 50$, $\bar{x} = 23.64$ ounces, and $s = .15$ ounces. The standard error is $s/\sqrt{n} = .15/\sqrt{50} = .02121$. So the confidence interval is

$$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} = 23.64 \pm 1.96 \times .02121 = 23.64 \pm .041578 = (23.598, 23.682)$$

Problem 5

This is a problem about proportions with $n = 100$ and $p = 78/100 = .78$. The standard error is $\sqrt{p(1-p)/n} = \sqrt{(.78)(.22)/100} = .0414$. So the confidence interval is

$$p \pm 1.96 \sqrt{\frac{p(1-p)}{n}} = .78 \pm 1.96 \times .0414 = .78 \pm .08115 = (.699, .861)$$

Problem 6

This is a problem about means and about finding a sample size. So the formula is

$$n = \left(\frac{1.96\sigma}{B} \right)^2$$

where σ is the population standard deviation, which we estimate by the sample standard deviation $s = 15$ in the preliminary sample, and where B is the desired half width (1). Thus $n = (1.96 \times 15/1)^2 = 864.36$. Round up to 865.