Stat 5102 Second Midterm Exam **April 13, 2001**

Name ______________________________ Student ID ____________________

The exam is open book, including handouts. It is closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 2 pages and 5 problems.

1. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. Beta$(\theta, 1)$, that is the densities are

$$f(x \mid \theta) = \theta x^{\theta - 1}, \qquad 0 < x < 1,$$

and suppose the prior distribution for θ is Exp(λ), where λ is a hyperparameter (a known constant).

   (a) Find the posterior distribution of θ.

   (b) Find the posterior mode of θ (note: *mode*, not mean or median).

2. [20 pts.] Suppose data X_1, $\ldots$, X_n are assumed to be i. i. d. with Normal$(\mu, 4)$, where μ is an unknown parameter. Suppose we want to do a Bayesian analysis with prior distribution for μ having prior distribution Normal$(0, 1)$.

   (a) Find the posterior distribution for μ, giving it either as a *normalized* probability density function or as a named distribution with specified values of the hyperparameters.

   (b) Find the 95% highest posterior density (HPD) region for μ.

3. [20 pts.] Let X_1, X_2, $\ldots$, X_n be i. i. d. discrete random variables having a distribution in the parametric family with probability functions

$$f(x \mid \lambda) = \left(1 - e^{-\lambda}\right) e^{-\lambda x}, \qquad x = 0, 1, \ldots$$

where $\lambda > 0$ is the parameter.

(a) Find the MLE of λ. You do not have to prove that your solution is the global maximizer of the likelihood. It is enough to find it.

(b) Find the expected Fisher information for λ.

(c) Give an asymptotic 95% confidence interval for λ.

4. [20 pts.] Suppose $X_1, \ldots, X_n$ are i. i. d. Normal(θ, θ^2), where θ is an unknown parameter, that is, θ is both the mean and standard deviation of each X_i.

   (a) Calculate the observed Fisher information $J_n(\theta)$.

   (b) Calculate the expected Fisher information $I_n(\theta)$.

5. [20 pts.] Suppose in a trial of a new cancer drug there were 500 subjects randomly assigned, 250 to the "treatment group" which received the new drug and 250 to the "control group" which received the standard therapy. The results of the trial were that 95 subjects in the treatment group died (38%) and 116 subjects in the control group died (46.4%).

   If p is the true population probability of death under treatment with the new drug and q is the true population probability of death under the standard therapy, conduct a test of the the hypotheses

$$\begin{aligned} H_0 &: p = q \\ H_A &: p < q \end{aligned}$$

   giving the P-value for the test.