Stat 5102 First Midterm Exam **March 2, 2001**

Name ______________________ Student ID ______________

The exam is open book, including handouts. It is closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 2 pages and 5 problems.

1. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\mathcal{N}(0, 4)$ and $\overline{X}_n$ and S_n^2 are the sample mean and variance for a sample of size $n = 9$ (with S_n^2 defined as in equation (7.17) in the notes with $n - 1$ in the denominator).

   Find $P(\overline{X}_n < S_n)$ exactly (not approximately).

2. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. from the distribution with density

$$f_\beta(x) = \frac{2 + 2(\beta - 1)x}{1 + \beta}, \qquad 0 < x < 1$$

   where $\beta > 0$ is an unknown parameter.

   Find a method of moments estimator for β.

3. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\text{Poi}(\mu)$. Because $E(X_i) = \text{var}(X_i) = \mu$, both the sample mean $\overline{X}_n$ and sample variance V_n are consistent estimators of μ.

   The fourth central moment of the $\text{Poi}(\mu)$ distribution is

$$E\{(X - \mu)^4\} = \mu + 3\mu^2.$$

   (a) Find the asymptotic distribution of $\overline{X}_n$. (Express all parameters in terms of μ.)

   (b) Find the asymptotic distribution of V_n. (Express all parameters in terms of μ.)

   (c) Find the asymptotic relative efficiency of $\overline{X}_n$ with respect to V_n considered as estimators of μ.

(d) State which is the better estimator.

4. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\text{Exp}(\lambda)$ random variables, where $\lambda > 0$ is an unknown parameter. As usual, let $\overline{X}_n$ denote the sample mean. What is the asymptotic distribution of

$$W_n = \frac{\overline{X}_n}{1 + \overline{X}_n}$$

5. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. $\text{Geo}(p)$ random variables, where p is an unknown parameter such that $0 < p < 1$ (and by $\text{Geo}(p)$ we mean the distribution described by Section B.1.8 of the Appendix on Brand Name Distributions in the notes). As usual, let $\overline{X}_n$ denote the sample mean. Find an asymptotic 95% confidence interval for p corresponding to data $n = 100$ and $\overline{X}_n = 4.73$.