Stat 5102 Second Midterm Exam **April 12, 2000**

Name ______________________________ Student ID ____________________

The exam is open book, including handouts. It is closed notes. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 2 pages and 5 problems.

1. [20 pts.] Suppose X_1, X_2, $\ldots$, X_n are i. i. d. discrete random variables with density

$$f_p(x) = \binom{x+m-1}{m-1} p^m (1-p)^x, \qquad x = 0, 1, 2, \ldots$$

where m is a known nonnegative integer constant and p is an unknown parameter satisfying $0 < p < 1$.

   (a) Find the MLE of p.

   (b) Find the observed or expected Fisher information p (either will do).

   (c) Find an asymptotic 95% confidence interval for p.

2. [20 pts.] Suppose X_1, X_2, $\ldots$ X_n are i. i. d. from the distribution with density

$$f(x) = \theta x^{-\theta-1}, \qquad x > 1,$$

where $\theta > 0$ is an unknown parameter. Suppose our prior distribution for the parameter θ is $\text{Exp}(\lambda)$, where λ is a known number (hyperparameter of the prior).

   (a) Find the posterior density of θ.

   (b) Find the posterior mean of θ.

3. [20 pts.] Suppose X_1, X_2, $\ldots$ are i. i. d. from a statistical model having a single parameter $\theta > 0$. I do not tell you anything about the model other

than that the MLE $\hat{\theta}_n$ exists and satisfies the conditions for the usual asymptotics to hold and that the Fisher information is

$$I_1(\theta) = \frac{\theta^2}{2}$$

(a) Find the Fisher information for the parameter $\varphi = \log(\theta)$.

(b) Find the asymptotic distribution of the MLE $\hat{\varphi}_n$ of the parameter φ.

4. [20 pts.] Suppose $X_1, X_2, \ldots$ are i. i. d. $\mathcal{N}(\mu, \sigma^2)$ distribution, where μ and σ^2 are unknown parameters, and suppose S_n^2 is the usual sample variance

$$S_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X}_n)^2.$$

Perform an exact test of the hypotheses

$$\begin{aligned} H_0 &: \sigma^2 \leq 1 \\ H_A &: \sigma^2 > 1 \end{aligned}$$

corresponding to sample size $n = 10$ and sample variance $S_n^2 = 2.3$. Give the P-value for the test and also say whether H_0 is accepted or rejected at the .05 level of significance.

5. [20 pts.] Suppose $X_1, X_2, \ldots$ are i. i. d. Geo(p) random variables and, as usual, $\overline{X}_n$ denotes the sample mean. Perform an asymptotic (large sample) test of the hypotheses

$$\begin{aligned} H_0 &: p = \tfrac{1}{4} \\ H_A &: p \neq \tfrac{1}{4} \end{aligned}$$

corresponding to sample size $n = 100$ and sample mean $\overline{X}_n = 3.6$. Give the P-value for the test and also say whether H_0 is accepted or rejected at the .05 level of significance.