Stat 5102 Final Exam **May 11, 2000**

Name ______________________ Student ID ______________

Secret Code ________________ (if you want your final grade posted).

The exam is open book, including handouts. It is closed notes. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 200. There are 3 pages and 8 problems.

1. [25 pts.] Suppose X_1, X_2, $\ldots$, X_n are i. i. d. $\text{Gam}(\alpha, \lambda)$ random variables and, as usual, $\overline{X}_n$ denotes the sample mean. What is the asymptotic distribution of $\log(\overline{X}_n)$? You must give the parameters of the asymptotic distribution as functions of α and λ for full credit.

2. [25 pts.] Suppose X_1, X_2, $\ldots$, X_n are i. i. d. $\text{Beta}(\theta, 1 - \theta)$ random variables, where $0 < \theta < 1$. Find a method of moments estimator of θ and its asymptotic distribution. You must give the parameters of the asymptotic distribution as functions of θ for full credit.

3. [25 pts.] Suppose X_1, X_2, $\ldots$, X_n are i. i. d. $\text{Beta}(s, t)$ random variables. Perform an asymptotic (large sample) test of the hypotheses

$$H_0 : s = t$$
$$H_A : s \neq t$$

corresponding to sample size $n = 100$, sample mean $\overline{X}_n = 0.57$, and sample variance $S_n^2 = 0.036$. Give the P-value for the test and also say whether H_0 is accepted or rejected at the .05 level of significance.

4. [25 pts.] Suppose X_1, X_2, $\ldots$, X_n are i. i. d. $\mathcal{N}(\mu, 25)$ random variables, and we observe $\overline{X}_n = 31.2$ for sample size $n = 16$. We want to do a Bayesian analysis with a $\mathcal{N}(20, 10)$ prior distribution for μ. Find a 95% HPD region for μ.

5. [25 pts.] In Problem 6-6 in the notes, the part of the posted solution was

```
xlow <- ifelse(x < 11, x - 11, 0)
xhig <- ifelse(x < 11, 0, x - 11)
out <- lm(y ~ xlow + xhig)
summary(out)
```

Recall that this fits a regression model with regression function

$$h(x) = \begin{cases} \alpha + \beta_1(x - 11), & x \le 11 \\ \alpha + \beta_2(x - 11), & x \ge 11 \end{cases}$$

Explain what two models are involved in the following printout.

```
> out.too <- lm(y ~ x)
> anova(out.too, out)
Analysis of Variance Table

Model 1: y ~ x
Model 2: y ~ xlow + xhig
  Res.Df Res.Sum Sq Df  Sum Sq F value   Pr(>F)
1     19    195.966
2     18    123.005  1  72.961  10.677 0.004277 **
```

Also explain why these are nested models and what conclusion about the fit of these two models can be drawn from the printout.

6. [25 pts.] Suppose X_1, X_2, $\ldots$, X_n are i. i. d. Beta$(\theta, 1)$ random random variables and the prior distribution of θ is Gam(α, λ). Find the posterior distribution of θ.

7. [25 pts.] Suppose we have regression data with variables `x` and `y` and fit a quadratic model

```
out <- lm(y ~ x + I(x^2))
options(show.signif.stars=FALSE)
summary(out)
```

getting the following (partial) output

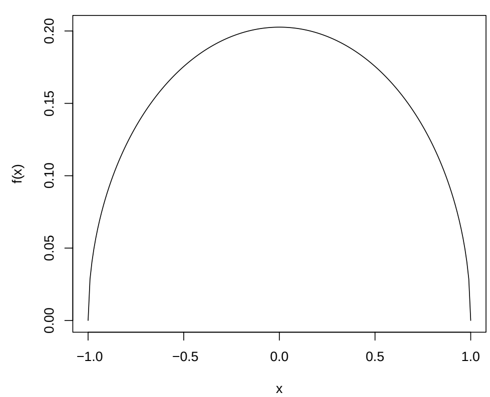
```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.544281   0.767327  -0.709    0.488
x            1.216011   0.168287   7.226 1.42e-06
I(x^2)      -0.011464   0.007784  -1.473    0.159

Residual standard error: 1.031 on 17 degrees of freedom
Multiple R-Squared: 0.9723,     Adjusted R-squared: 0.969
```

Give a 90% confidence interval for the coefficient of x^2 in the regression function.

8. [25 pts.] Suppose $X_1, X_2, \ldots, X_n$ are i. i. d. random variables with density

$$f(x) = \frac{2}{\pi}\sqrt{1 - x^2}, \qquad -1 < x < +1$$

shown below



What is the asymptotic distribution of the sample median?