

Stat 5101 (Geyer) Fall 2020
Homework Assignment 12
Due Wednesday, December 16, 2020

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

12-1. Give the details of the argument that the $\text{Poi}(\mu)$ distribution is approximately normal when μ is large.

12-2. Suppose X_1, X_2, \dots are IID with mean μ and variance σ^2 and

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

What is the approximate normal distribution of $\sin(\bar{X}_n)$ when n is large?

12-3. Suppose X_1, X_2, \dots are IID $\text{Poi}(\mu)$ random variables and

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

To what random variable does

$$\sqrt{n}(e^{-\bar{X}_n} - e^{-\mu})$$

converge in distribution?

12-4. Suppose X_1, X_2, \dots are IID $\text{Ber}(p)$ random variables with $0 < p < 1$ and

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

(a) What is the approximate normal distribution of $\bar{X}_n(1 - \bar{X}_n)$ when n is large?

(b) There is something unusual about the case $p = 1/2$. What is that?

12-5. Suppose X is a $\text{Poi}(\mu)$ random variable. For what function g does $g(X)$ have approximate normal distribution for large μ with variance that is a constant function of the parameter?

12-6. Suppose X_1, X_2, \dots are IID $\text{Exp}(\lambda)$ random variables and

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

For what function g does $g(\bar{X}_n)$ have approximate normal distribution for large n with variance that is a constant function of the parameter?

12-7. Suppose X_1, X_2, \dots , is an IID sequence of random variables, having four ordinary moments

$$\alpha_i = E(X_n^i), \quad i = 1, \dots, 4.$$

Define

$$Y_n = X_n^2, \quad n = 1, 2, \dots$$

and

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$
$$\bar{Y}_n = \frac{1}{n} \sum_{i=1}^n Y_i$$

What is the approximate normal distribution of $\bar{Y}_n - \bar{X}_n^2$ when n is large?
Hint: Slides 93–95, deck 7 and the multivariate delta method.

Review Problems from Previous Tests

12-8. Suppose X_1, X_2, \dots are IID $\text{Geo}(p)$ random variables and

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

What is the approximate normal distribution of

$$\frac{1}{1 + \bar{X}_n}$$

when n is large?

12-9. Suppose X is a $\text{chi}^2(n)$ random variable. What is the variance stabilizing transformation: for what function g does $g(X)$ have approximate normal distribution for large n with variance that is a constant function of the parameter n ?