Stat 5101 (Geyer) Fall 2020
Homework Assignment 11
Due Wednesday, December 9, 2020

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

11-1. Suppose the random vector (X, Y) has variance matrix

$$\begin{pmatrix} 4 & -3 \\ -3 & 9 \end{pmatrix}$$

Find $\operatorname{sd}(X)$, $\operatorname{sd}(Y)$, and $\operatorname{cor}(X, Y)$. Note: $\operatorname{cor}(X, Y)$ is correlation not covariance.

11-2. Suppose $\operatorname{sd}(X) = 5$, $\operatorname{sd}(Y) = 7$, and $\operatorname{cor}(X, Y) = 2/3$. Find the variance matrix of the random vector (X, Y).

11-3. Suppose X has the Poisson distribution with mean 100.

(a) Calculate $\Pr(X < 80)$ exactly.

(b) Calculate $\Pr(X < 80)$ using the normal approximation without correction for continuity.

(c) Calculate $\Pr(X < 80)$ using the normal approximation with correction for continuity.

(d) Which of (b) and (c) is closer to correct?

(e) Calculate $\Pr(X > 120)$ exactly.

(f) Calculate $\Pr(X > 120)$ using the normal approximation without correction for continuity.

(g) Calculate $\Pr(X > 120)$ using the normal approximation with correction for continuity.

(h) Which of (f) and (g) is closer to correct?

Be careful about weak and strict inequality.

11-4. Suppose X_1, ..., X_{40} are IID random variables having the exponential distribution with rate parameter one. Let $Y = X_1 + \cdots + X_{40}$.

(a) Calculate $\Pr(Y < 25)$ exactly.

(b) Calculate $\Pr(Y < 25)$ using the normal approximation.

(c) Calculate $\Pr(Y > 55)$ exactly.

(d) Calculate $\Pr(Y > 55)$ using the normal approximation.

11-5. Suppose $X_1, \ldots, X_{50}$ are IID random variables having mean 10 and standard deviation 5. Let $\overline{X}_n = (X_1 + \cdots + X_n)/n$.

(a) Calculate $\Pr(\overline{X}_n < 9)$ using the normal approximation.

(b) Calculate $\Pr(\overline{X}_n > 11)$ using the normal approximation.

11-6. Suppose $X_1, \ldots, X_n$ are IID random variables having mean μ and standard deviation $\sigma > 0$. Let $\overline{X}_n = (X_1 + \cdots + X_n)/n$. Find a number c, which will be a function of σ and n, such that

$$\Pr(|\overline{X}_n - \mu| > c) \approx 0.05,$$

where the $\approx$ means approximately equal using the normal approximation.

11-7. Suppose $X_1, X_2, \ldots$ is a sequence of IID random variables having mean μ and standard deviation $\sigma > 0$. Let $\overline{X}_n = (X_1 + \cdots + X_n)/n$. Does

$$\frac{n(\overline{X}_n - \mu)^2}{\sigma^2}$$

converge in distribution? If so, to what distribution does it converge? Hint: CLT and continuous mapping theorem.

11-8. Suppose $X_1, X_2, \ldots,$ is a sequence of random variables, θ is a constant, and

$$\sqrt{n}(X_n - \theta) \xrightarrow{\mathcal{D}} Y$$

where Y is any random variable. Show that this implies

$$X_n \xrightarrow{P} \theta.$$

Hint: Slutsky's theorem.

11-9. Suppose $X_1, X_2, \ldots,$ is a sequence of IID random variables, having mean μ and standard deviation $\sigma > 0$. Suppose

$$S_n = g_n(X_1, \ldots, X_n)$$

is some function of the data such that

$$S_n \xrightarrow{P} \sigma.$$

Show that

$$\frac{\sqrt{n}(\overline{X}_n - \mu)}{S_n} \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1).$$

Hint: CLT and Slutsky's theorem.