Stat 5101 (Geyer) Fall 2020
Homework Assignment 3
Due Wednesday, September 30, 2020

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

3-1. Suppose that f is a PMF on a sample space S, suppose X and Y are random variables in this probability model. Prove the following statements.

(a) $E(X + Y) = E(X) + E(Y)$.

(b) If $X(s) \geq 0$ for all $s \in S$, then $E(X) \geq 0$.

(c) If $Y(s) = a$ for all $s \in S$, then $E(XY) = aE(X)$.

(d) If $Y(s) = 1$ for all $s \in S$, then $E(Y) = 1$.

Do not use the axioms (these are the axioms). The problem is to prove that these statements follow from our earlier definition of PMF and expectation.

3-2. Suppose X has the uniform distribution on the set $\{1, 2, 3, 4\}$, and suppose $Y = X^2$.

(a) Calculate $E(X)$.

(b) Calculate $E(Y)$.

(c) Calculate $E(Y/X)$.

(d) Calculate $E(Y)/E(X)$.

(e) Compare your answers in (c) and (d). Are they the same? Should they be the same?

3-3. Suppose $E(X) = 3$ and $E(Y) = 4$. Calculate $E(5X + Y)$.

3-4. Suppose X is a random variable having PMF given by

x	1	2	3	4	5
$f(x)$	1/9	2/9	3/9	2/9	1/9

(a) Calculate $E(X)$.

(b) Calculate $\text{var}(X)$.

3-5. Suppose X is a Ber(p) random variable and $Y = 2X - 1$.

(a) Calculate $E(Y)$.

(b) Calculate $\text{var}(Y)$.

(c) Calculate $E(Y^2)$.

(d) Calculate $\text{var}(Y^2)$.

3-6. Suppose X has the discrete uniform distribution on the set

$$\{ x \in \mathbb{Z} : l \le x \le u \}$$

where l and u are integers with $l < u$.

(a) Calculate $E(X)$.

(b) Calculate $\text{var}(X)$.

3-7. Suppose X and Y are random variables in the same probability model, and suppose a, b, c, and d are constants.

(a) Prove that

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

(b) Prove that

$$\text{cov}(a + bX, c + dY) = bd \, \text{cov}(X, Y)$$

3-8. Suppose

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

(a) Calculate $\mathbf{AB}$.

(b) Calculate $\mathbf{BA}$.

(c) Compare your answers in (a) and (b). Are they the same? Should they be the same?

3-9. Suppose X is a random variable with mean μ and variance σ^2. Calculate the mean vector and variance matrix of the random vector $\mathbf{Y} = (X, 2 + 3X)$.

3-10. Suppose $\mathbf{X}$ is a random variable with mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} v_1 & c \\ c & v_2 \end{pmatrix}$$

Calculate the mean vector and variance matrix of the random vector $\mathbf{Y} = \mathbf{a} + \mathbf{B}\mathbf{X}$ where

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

Review Problems from Previous Tests

3-11. Suppose X is a random variable having probability mass function (PMF) given by

x	-2	-1	0	1	2
$f(x)$	$1/3$	$1/6$	0	$1/6$	$1/3$

(a) Calculate $E(X)$.

(b) Calculate $\text{var}(X)$.

3-12. Suppose X is a random variable having PMF given by

x	-2	-1	0	1	2
$f(x)$	$1/9$	$2/9$	$1/3$	$2/9$	$1/9$

(a) Find the PMF of the random variable $Y = X^2$.

(b) Calculate $\Pr(Y > 0)$.

3-13. Suppose the random vector (X, Y) has PMF given by

$$f(x, y) = \frac{x^2 y}{90}, \qquad x = -2, -1, 0, 1, 2, \ y = 2, 3, 4.$$

Are X and Y independent random variables? Explain why or why not, as the case may be.