Stat 5101 (Geyer) Fall 2020
Homework Assignment 2
Due Wednesday, September 23, 2020

Solve each problem. Explain your reasoning. No credit for answers with no explanation.

2-1. Suppose we have a PMF f_X with domain S (the original sample space), and we have a map $g : S \to T$ that induces a probability model with PMF f_Y with domain T (the new sample space) given by the formula on slide 82. Prove that for any real-valued function h on T

$$\sum_{y \in T} h(y) f_Y(y) = \sum_{x \in S} h\bigl(g(x)\bigr) f_X(x)$$

2-2. Suppose we have the uniform distribution on $S = \{-2, -1, 0, 1, 2\}$ and the random variable X defined by

$$X(s) = s^2, \qquad s \in S.$$

Determine the PMF of the random variable X.

2-3. Suppose $\mathbf{X} = (X_1, X_2)$ has the uniform distribution on $\{1, 2, 3, 4, 5, 6\}^2$.

(a) Show that the components of $\mathbf{X}$ are independent.

(b) Determine the PMF of the distribution of the random variable $Y = X_1 + X_2$.

(c) Determine $E(Y)$.

(d) Define $\mu = E(Y)$. Determine $E\{(Y - \mu)^2\}$.

2-4. Suppose $\mathbf{X} = (X_1, X_2, X_3)$ has the uniform distribution on the set

$$\{(0, 0, 0), (1, 1, 0), (1, 0, 1), (0, 1, 1)\}$$

Show that the components of $\mathbf{X}$ are pairwise independent — meaning (X_1, X_2), (X_2, X_3) and (X_3, X_1) each have independent components — but not independent.

2-5. Define $S = \{1, 2, 3, 5, 6\}$. Each of the following functions is the PMF of a random vector. For each say whether the components of that random vector are independent or dependent, and say why.

(a) $f(x, y) = (x + y)/170$, $(x, y) \in S^2$.

(b) $f(x, y) = xy/289$, $(x, y) \in S^2$.

(c) $f(x, y) = (5x)/(187y)$, $(x, y) \in S^2$.

2-6. Suppose $\mathbf{X} = (X_1, X_2)$ has the uniform distribution on the set $\{-1, 0, 1\}^2$. Define $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

(a) Determine the PMF of the distribution of Y_1.

(b) Determine the PMF of the distribution of Y_2.

(c) Determine the PMF of the distribution of $\mathbf{Y} = (Y_1, Y_2)$.

(d) Show that the components of $\mathbf{X}$ are independent.

(e) Show that the components of $\mathbf{Y} = (Y_1, Y_2)$ are independent or not independent, whichever is correct.

2-7. A standard deck of playing cards has 52 cards. How many arrangements of the 52 cards are there? (You may give your answer symbolically, you don't have to write out this large number.)

2-8. In the game of poker, played with a standard deck of playing cards, a hand has 5 cards and the order in which the cards are dealt doesn't matter. How many different poker hands are there? (You may give your answer symbolically, you don't have to write out this large number.)

2-9. A standard deck of playing cards, the cards have 13 different ranks and 4 different suits, each card having one rank and one suit. In the game of poker, a hand is said to have one pair, if two cards (the "pair") have the same rank and the other three cards have different ranks (different from each other and different from the pair). How many different poker hands have one pair? (You may give your answer symbolically, you don't have to write out this large number.)

Hint: It helps to think of this problem in stages. First you choose the rank of the pair. Second you choose the ranks of the other three cards. Third you choose which 2 cards (from the four suits) the pair will be. Fourth you chose which card (from the four suits) each of the non-pair cards will be.

2-10. Suppose X has the $\text{Bin}(n, p)$ distribution. Determine $E\{X(X-1)\}$.