Stat 5101 (Geyer) Fall 2020
Homework Assignment 1
Due Wednesday, September 16, 2020

Solve each problem. Explain your reasoning. No credit for answers with no explanation.

1-1. For each of the following functions h either determine a constant c such that $c \cdot h$ — that is, the function $x \mapsto c \cdot h(x)$ — is a PMF or determine that no such constant exists.

(a) the identity function on the set $\{0, 1, 2\}$.

(b) the identity function on the set $\{-2, -1, 0, 1, 2\}$.

(c) the constant function $x \mapsto 1$ on the set $\{0, 1, 2\}$.

(d) the constant function $x \mapsto 1$ on the set $\{-2, -1, 0, 1, 2\}$.

(e) the function $x \mapsto x^2$ on the set $\{0, 1, 2\}$.

(f) the function $x \mapsto x^2$ on the set $\{-2, -1, 0, 1, 2\}$.

(g) the function $x \mapsto x^3$ on the set $\{0, 1, 2\}$.

(h) the function $x \mapsto x^3$ on the set $\{-2, -1, 0, 1, 2\}$.

1-2. Suppose X is a random variable having the discrete uniform distribution on the sample space $\{1, 2, 3, 4, 5, 6\}$.

(a) Determine $\Pr(X < 4)$.

(b) Determine $\Pr(X \le 4)$.

(c) Determine $\Pr(6 < X < 10)$.

1-3. Suppose X is a random variable having PMF

$$f(x) = \frac{x}{21}, \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Determine $E(X)$.

(b) Determine $E(X^2)$.

(c) Determine $E\{(X - 3)^2\}$.

1-4. Suppose X is a Ber(p) random variable.

(a) Show that $E(X^k) = p$ for all positive integers k.

(b) Determine $E\{(X - p)^2\}$.

(c) Determine $E\{(X - p)^3\}$.

1-5. Determine the set of real numbers θ such that

$$f_\theta(x) = \begin{cases} \theta, & x = x_1 \\ \theta^2, & x = x_2 \\ 1 - \theta - \theta^2, & x = x_3 \end{cases}$$

is a PMF on the sample space $\{x_1, x_2, x_3\}$.