Stat 8112 (Geyer) Spring 2013
Homework Assignment 6
Due Friday April 12, 2013

6-1. Show that the regularity conditions (a) through (e) of Appendix C the preprint about "no-n" asymptics hold for an exponential family having parameter space $\mathbb{R}^d$ in a minimal natural parameterization. In verifying (e) use the section "passing the derivative under the integral sign" on p. 124 of Ferguson.

6-2. Same as Problem 1 except the model is the Cauchy location family.

6-3. Same as Problem 1 except the model is the Cauchy location-scale family.

6-4. Show that the general Pearson chi-square statistic, the top displayed equation on p. 58 in Ferguson except that p_j, the vector of probabilities in the point null hypothesis in Ferguson, is replaced by $p_j(\theta^*)$, the vector of probabilities corresponding to the MLE in a compound null hypothesis, is either a Rao or Wald test statistic (one or the other, figuring out which is part of the problem — it obviously isn't Wilks because there are no logs in the formula). What conditions are required on the geometry of the null hypothesis? What is the alternative hypothesis?

6-5. Most intro statistics tests now scorn the so-called "exact" t-test for comparing the means of two independent samples from two different populations that assumes exact normality of the populations and exact equality of population standard deviations. Instead they teach the Welch approximate t-test that uses the same test statistic as the "large sample" z-test (see my 5102 slides)

`http://www.stat.umn.edu/geyer/5102/slides/s2.pdf`

slides 134–146. This is also the default procedure done by the R function `t.test`. This problem, however, is not about the Welch approximation. It is about generalizing it to three or more populations.

Assume we have IID samples, which are also independent of each other, from different populations and we assume normality of all populations (we do not assume equality of population standard deviations). The null hypothesis is all population means equal. The alternative hypothesis is falsity of the null hypothesis. These are exactly the standard assumptions for one-way ANOVA except that the standard assumptions also assume homoscedasticity (equality of population standard deviations) and we do not. What is the Wald test statistic for this situation? Do enough work trying to find the MLE under the null hypothesis to convince yourself it has no closed-form expression.