Stat 8112 (Geyer) Spring 2013
Homework Assignment 4
Due Monday March 11, 2013

4-1. Show the following

(a) $O_p(X_n)O_p(Y_n) = O_p(X_nY_n)$.

(b) $O_p(X_n)o_p(Y_n) = o_p(X_nY_n)$.

(c) $o_p(X_n)o_p(Y_n) = o_p(X_nY_n)$.

(d) If $X_n = o_p(1)$, then $o\big(O_p(X_n)\big) = o_p(X_n)$.

You may assume that all random elements involved in this problem are real-valued (multiplication of vectors is undefined). These problems are to show that the left-hand side can be replaced by the right-hand side, that is, for (a), if $U_n = O_p(X_n)$ and $V_n = O_p(Y_n)$, then $U_nV_n = O_p(X_nY_n)$.

4-2. A problem unrelated to anything (came up in research, just want to see what everyone does). Suppose X is a random element of $\mathbb{R}^d$ having PDF f_θ and define $Y = I_A(X)$, where A is some measurable subset of $\mathbb{R}^d$. This problem is about the joint distribution of the pair (X, Y) and its marginals and conditionals. Since X is continuous and Y is discrete, this is a bit tricky. In measure-theoretic terms you can give a density with respect to the product of Lebesgue measure on $\mathbb{R}^d$ and counting measure on $\{0, 1\}$. If you haven't had measure theory, you can still do this just intuitively treating X as continuous and Y as discrete. If g_θ is the joint density of X and Y (that we are trying to find) then the expectation of any function h is given by

$$\sum_{y=0}^{1} \int h(x, y) g_\theta(x, y)\, dx$$

(integrate over the continuous variable and sum over the discrete variable).

(a) Give the joint density g_θ of the pair (X, Y).

(b) Give the marginal density of X and the conditional density of Y given X.

(c) Give the marginal density of Y and the conditional density of X given Y.

4-3. Ferguson, Problem 9-2.

4-4. Ferguson, Problem 10-1.

4-5. Ferguson, Problem 10-2.

4-6. Ferguson, Problem 13-1.

4-7. Ferguson, Problem 13-2.

4-8. Ferguson, Problem 13-4.