STAT4102 practice final, Spring 2008

1. You've taken five observations from a Gamma$(2, \theta)$ distribution; they are 3.5, 4.3, 0.6, 1.1, and 2.2. Test $H_0 : \theta = 0.25$ against $H_a : \theta \neq 0.25$ using the likelihood ratio test.

2. You have one observation X that is limited to values between 0 and 1. You wish to test $H_0 : f_X(x) = 1$ against $H_a : f_X(x) = 2x$. Find the rejection region and power of the most powerful test with $\alpha = 0.1$. Using this test, what would you conclude if $X = 0.7$?

3. A manufacturer of light bulbs claims that its light bulbs have a mean life of 1520 hours. A random sample of 40 such bulbs is selected for testing. If the sample produces a mean value of 1498.3 hours and a standard deviation of 85 hours, is there sufficient evidence to claim that the mean life is significantly less than the manufacturers claim, using the $\alpha = .01$ significance level? State the hypotheses, report the p-value, and draw the appropriate conclusion in context.

   Assuming the true standard deviation is really 85 hours, how large of a difference could this study detect with a power of 0.5 or greater?

   When you look more closely at the data, you find that the data is not normally distributed, but has a few large values as outliers, so you decide to do a sign test. You count that 30 of the bulbs have a life of less than 1520 hours. State the hypotheses, report the p-value, and draw the appropriate conclusion in context.

4. For a random sample of size n from an Exponential distribution with rate parameter λ (so that the density is $f_Y(y) = \lambda e^{-\lambda y}$), derive the maximum likelihood estimator, the methods of moments estimator, and the Bayes estimator (that is, the posterior mean) using a prior proportional to $\lambda e^{-\lambda}$, for $\lambda > 0$. (Hint: the posterior distribution will be a Gamma.)

5. For a random sample of size n from a Poi(λ) distribution. Find the MSE of the sample mean by finding the bias and the variance. Is the sample mean consistent for λ? Write out the likelihood function. Is the sample mean sufficient for λ? Finally, determine if the sample mean is efficient by calculating the Cramer-Rao lower bound.