STAT4101 Fall 2007 Practice Exam 2. You are permitted two sheets of paper with notes, front and back, and a calculator. You will be given a copy of the normal table and the table of distributions from the inside covers of your book.

1. Let the random variable X have a probability density function (pdf) of

$$f(x) = \begin{cases} c(1-x^2) & \text{for } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find c.

   (b) Find the cumulative distribution function (cdf) of X, $F(x)$.

   (c) Find $P(X > 0)$.

   (d) Find $E\left(\frac{1}{1-x^2}\right)$.

2. Suppose scores on a certain entrance exam are approximately normal with mean 75 and standard deviation 10.

   (a) School A sets the cutoff for admission at 68. What percent of students are admitted?

   (b) Given that a particular student was admitted to school A, what's the probability that their score was above 75?

   (c) School B wants to set their cutoff for admission so that only 25% of students are admitted. What should the cutoff be?

3. Suppose the random variable X has moment generating function of

$$m_X(t) = e^{3t+2t^2}.$$

   (a) Identify the distribution of X.

   (b) We know that if $Y = aX + b$, $m_Y(t) = e^b m_X(at)$.
   What is the moment generating function of $Y = \frac{X}{2} - 3$?

   (c) Identify the distribution of Y.

4. Let X and Y have the following joint probability function:

		Y		
		1	2	3
X	1	0.08	0.1	0.22
	2	0.12	0.2	0.28

   (a) Find $P(X = 1, Y \leq 2)$.

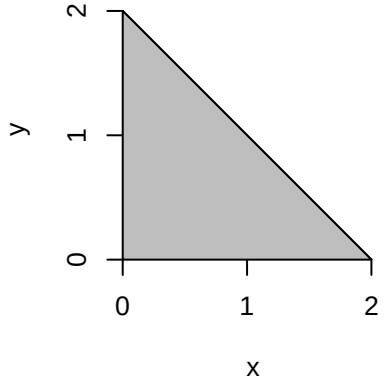
   (b) Find the marginal density of X.

   (c) Find the conditional density of $X|Y = 1$.

   (d) Find $E(X|Y = 1)$.

   (e) Are X and Y independent? Why or why not?

5. Let X and Y be uniform over the region



so it has pdf

$$f(x,y) = \begin{cases} \frac{1}{2} & \text{for } x > 0, y > 0, x + y < 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(Y < 1)$.

(b) Find $P(Y < 1 | X = 1)$.

(c) Find the marginal density of X.

(d) Find the conditional density of $Y|X$.

(e) Find $E(Y|X)$.

(f) Are X and Y independent? Why or why not?

6. Let X and Y have joint density

$$f(x,y) = c\frac{x}{y+1}$$

where c is the necessary constant.

(a) Find the marginal density of Y (in terms of c).

(b) Find the conditional density of $X|Y$ (in terms of c).

(c) Find $E(Y + 1)$ (in terms of c).

(d) Set up the integral to find $P(X < Y)$.

(e) Are X and Y independent? Why or why not?

7. Let $E(X) = 5$, $\operatorname{Var} X = 4$, $E(Y) = 3$, $\operatorname{Var} Y = 1$, and $E(XY) = 16$.

(a) Find the correlation between X and Y.

(b) Find $E(2X - 3Y + 4)$.

(c) Find $\operatorname{Var}(2X - 3Y + 4)$.

8. Let $X|Y$ be Binomial with $n = 100$ and $p = Y$, and let Y be Beta with $\alpha = 99$ and $\beta = 1$. Find $E(X)$.