1. **(2.57)** Two events, A and B, are such that $P(A) = .5$, $P(B) = 0.3$, and $P(A \cap B) = .1$. Find the following:
   (a) $P(A|B)$
   (b) $P(B|A)$
   (c) $P(A|A \cup B)$
   (d) $P(A|A \cap B)$
   (e) $P(A \cap B|A \cup B)$

2. **(2.71)** Two events, A and B, are such that $P(A) = .2$, $P(B) = 0.3$, and $P(A \cup B) = .4$. Find the following:
   (a) $P(A \cap B)$
   (b) $P(\overline{A} \cup \overline{B})$
   (c) $P(\overline{A} \cap \overline{B})$
   (d) $P(\overline{A}|B)$

3. **(2.72)** Two events, A and B, are such that $P(A) = .5$, $P(B) = 0.2$, and A and B are independent. Find the following:
   (a) $P(A \cup B)$
   (b) $P(\overline{A} \cap \overline{B})$
   (c) $P(\overline{A} \cup \overline{B})$

4. **(2.62)** Show that if any of following three equalities from the definition of independence hold, they all hold. That is, first assume the first is true and prove the other two. Then repeat for the second and third. If you like, you can use results proved in earlier parts.
   (a) $P(A \cap B) = P(A)P(B)$
   (b) $P(A|B) = P(A)$
   (c) $P(B|A) = P(B)$

5. **(2.65)** If $P(A > 0)$, $P(B > 0)$, and $P(A) < P(A|B)$, show that $P(B) < P(B|A)$.

6. **(2.66)** If A, B, and C are three events such that $P(A \cap B) = P(A \cap C) \neq 0$ but $P(B \cap C) = 0$, show that
$$P(\text{at least one of the three events occuring}) = P(A) + P(B) + P(C) - 2P(A \cap B).$$
   Drawing a Venn diagram will help you understand what's happening, but you should also show this algebraically. Hints: Describe what "at least one of the three events occuring" means in set notation, and remember that intersections and unions are distributive (see page 24).

7. **(2.60)** A survey of customers in a particular community showed that 10% were dissatisfied with plumbing jobs done in their homes. Half the complaints dealt with plumber A, who does 40% of the plumbing jobs in the town. Using set notation,
   (a) find the probability that a customer will obtain an unsatisfactory plumbing job, given that the plumber was A.
   (b) find the probability that a customer will obtain an satisfactory plumbing job, given that the plumber was A.

8. **(2.78)** Diseases I and II are prevalent among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease II eventually, and 3% will contract both diseases. Using set notation,
   (a) find the probability that a randomly chosen person from this population will contract at least one disease.
   (b) find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.

9. **(2.87)** An advertising agency notices that approximately 1 in 50 potential buyers of a product sees a given magazine ad, and 1 in 5 sees a corresponding ad on television. One in 100 sees both. One in 3 purchases the product after seeing (at least one) ad, 1 in 10 without seeing either ad. Using set notation, find the probability that a randomly selected potential customer will purchase the product?

10. **(2.99)** A diagnostic test for a disease is said to be 90% accurate in that if a person has the disease, the test will detect it with probability 0.9. Also, if a person does not have the disease, the test will report that he or she does not have it with probability 0.9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease. what is the conditional probability that she does, in fact, have the disease? (Please use set notation.) Are you surprised by the answer? Would you call this diagnostic test reliable?

11. *(from 2.88 and 2.89)* Three radar sets, operating independently, are set to detect any aircraft flying through a certain area. Each set has a 0.98 probability of detecting an aircraft in this area. Find the following probabilities, writing out the events of interest using set notation.
    (a) If an aircraft enters the area, what is the probability that it is detected by at least one of the radar sets?
    (b) If an aircraft enters the area, what is the probability that it is detected by all three of the radar sets?
    (c) If three aircraft enter the area, what is the probability that at least one is not detected? Assume the detection of different aircrafts are independent.

12. **(2.115)** Five identical bowls are labeled 1, 2, 3, 4, and 5. Bowl i contains i white and $5 - i$ black balls, with $i = 1, 2, \ldots 5$. A bowl is randomly selected and two balls are randomly selected (without replacement) from the contents of the bowl. Using set notation,
    (a) What is the probability that both balls selected are white?
    (b) Given that both balls selected are white, what is the probability that bowl 3 was selected?

These problems are either taken directly from, or inspired by, our text.