1. What is the distribution of X (or (X, Y)) in each of the following situations? If the distribution has a name, please state it, and if it has parameters, please state those also. If the distribution has a shorthand notation, you may use that, for example, $X \sim \text{Bin}(5, 0.4)$. (In some situations, the distribution may not be exact. In that case, choose the distribution that best fits the situation.)

   a) A machine produces parts one at at a time, and each part is judged to be either good or defective. The probability of a defective part is 0.1. The operator needs to produce 100 good parts to fill his quota for the day. Let X be the total number of parts he makes in order to fill his quota.

   b) Same machine and operator as above. The boss stops by to watch, and stays while 10 parts are made. Let X be the number of bad parts the boss sees made.

   c) Flaws occur in a rope at random, but on average are five feet apart. Let X be the number of flaws in a fifty foot piece of rope.

   d) A box of 100 nails has 5 that are defective. A carpenter takes 10 out of the box at random to make his next project. Let X be the number of defective nails that he took.

   e) Customers come to a certain post office either to send a package or pick up their mail (but never both). On average, someone comes in to mail a package every ten minutes, and to pick up their mail every five minutes. Let X be the total number of customers that enter the post office between noon and 1pm.

   f) A person drops a ball into a box that is one foot long and one foot wide. Assume the ball is equally likely to be anywhere in the box, and let (X, Y) denote the position of the ball in the box.

   g) John says meteors hit the earth on average every 100 years. Assuming John is right, let X be the number of years until the next meteor hits earth.

   h) 49 Democrats, 49 Republicans, and 2 Independents are in the Senate. We choose a senator at random. Let X be the party of this senator.

2. Let $X \sim \text{Bin}(10, 0.4)$. Find $P(X = 2)$. Find $P(X \leq 2)$. Find $E(X)$. (Copies of p. 648 and p. 650 will be included in the exam if a problem like this is included.)

3. Let $X \sim \text{Geo}(p)$ and $Y \sim \text{NegBin}(3, p)$, where X and Y are independent. Find $P(X + Y = 0)$. Find the mean and variance of $X + Y$.

4. Suppose that some event occurs according to a Poisson process, with 2 events per minute on average. What's the average length of time until the next event? What's the probability of no events in the next minute?

5. Suppose calls come into a service center following a Poisson process, average 10 calls an hour. Let X be the number of calls an operator answers in a given hour, so $X \sim \text{Poi}(10)$. With probability 0.1, the operator cannot answer the question and must forward it to the supervisor. What's the expected number of calls an operator must forward to the supervisor in a given hour? Use the following steps to help you.

   a) Assume we already know how many total calls were received (X), let Y be the number of calls that were forwarded. What's the distribution of Y, given X known?

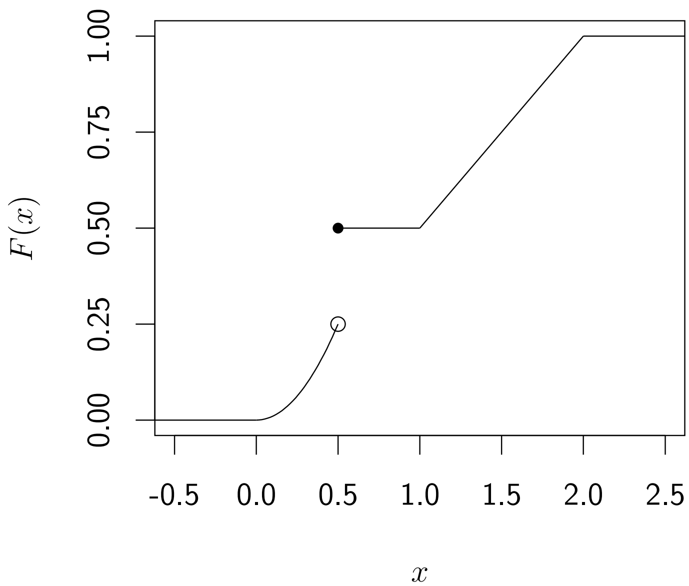
   b) What's the expectation of Y given X known?

   c) Now find $E(Y)$. Remember $E(Y) = E(E(Y|X))$.

6. Let the cumulative distribution function of X be

$$F(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x^2 & \text{for} \quad 0 \leq x < 1/2 \\ 1/2 & \text{for} \quad 1/2 \leq x < 1 \\ x/2 & \text{for} \quad 1 \leq x < 2 \\ 1 & \text{for} \quad x > 2, \end{cases}$$

as in the following plot.



a) Find $P(X < 1/4)$.
b) Find $P(X = 1/2)$.
c) Find $P(X > 1 | X > 1/2)$.
d) Let $Y = 2X$. What's the cumulative distribution function of Y?

7. The probability density function for X is $f(x) = c(1 - x^2)$ for $-1 < x < 1$ and 0 otherwise.

a) Show $c = 3/4$.
b) Find the mean and median of X.
c) Find $P(X = 1/2)$.
d) Find F and use it to find $P(0 < X < 1/2)$.
e) Find the density of $2X$.

8. Let $X \sim \text{Unif}(0, 1)$ and $Y \sim \text{Exp}(1)$, independently.

a) What's the joint density of X and Y? Remember to state the support.
b) Find $E(XY)$.
c) Find $P(Y < X)$.

9. Let the joint density of X and Y be uniform over the unit circle.
Are X and Y independent? Why or why not?

10. $P(X = 0) = 1/2$, $P(X = 1) = 1/4$, and $P(X = 2) = 1/4$. Use the pgf to find $\text{Var}(X)$.