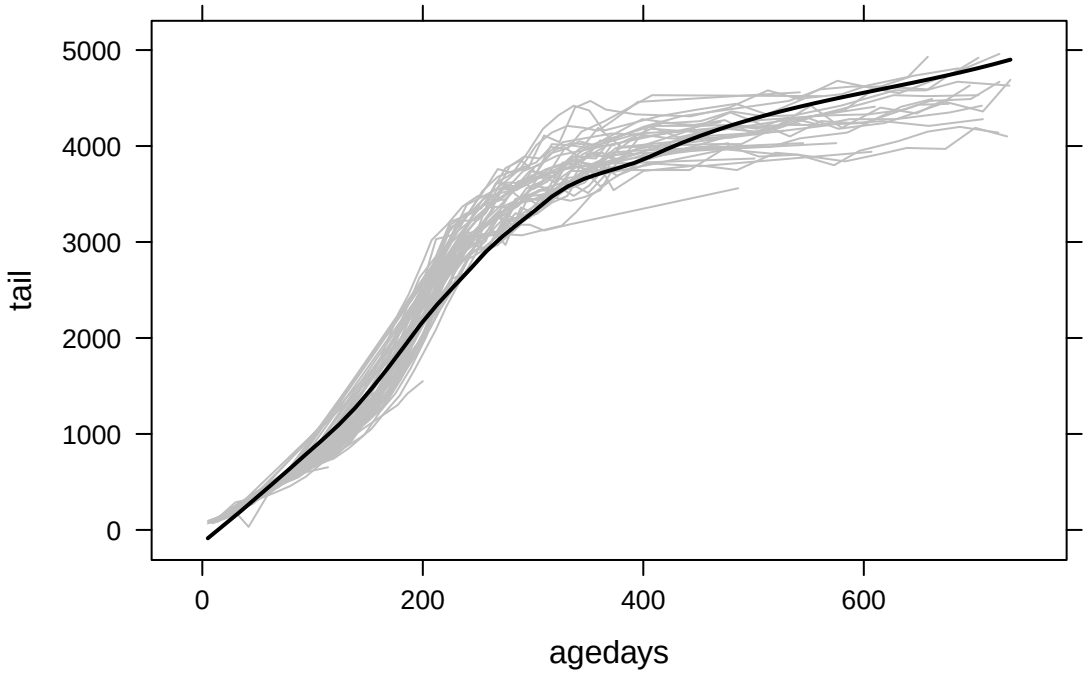
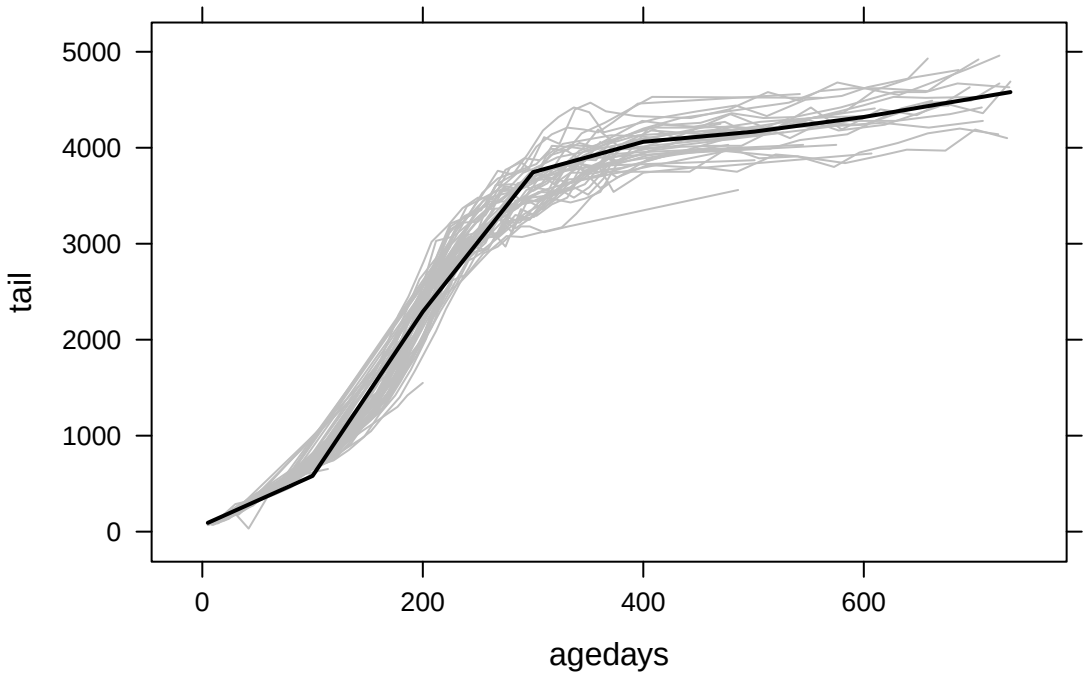
Wallaby tail data

	animal	sex	agedays	tail
1	45	1	14	93
2	45	1	28	185
3	45	1	49	307
4	45	1	69	454
5	45	1	83	568
6	45	1	97	648

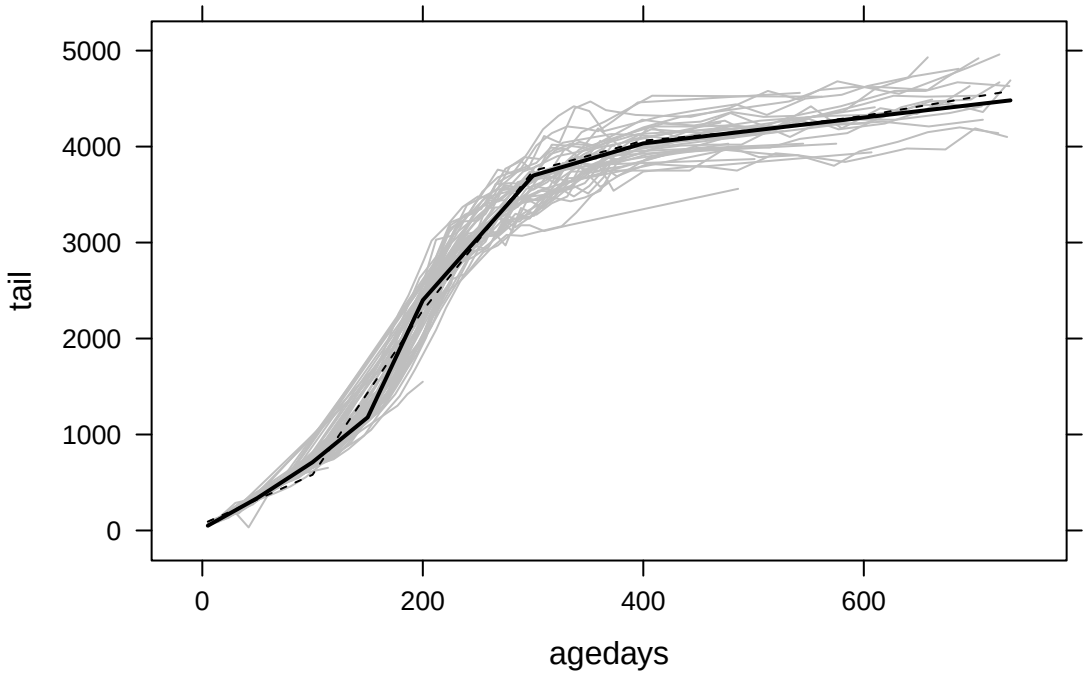


Linear Piecewise

```
> knot <- function(t, x, degree = 1) {
+     (x - t)^degree * (x > t)
+ }
> knots <- function(t, x, degree = 1) {
+     names(t) <- paste("knot", t, sep = ".")
+     sapply(t, knot, x = x, degree = degree)
+ }
> m.l1 <- lme(tail ~ agedays + knots((1:6) * 100, agedays), random = ~agedays |
+     animal, data = wal)
> summary(m.l1)

...
                     Value Std.Error   DF   t-value p-value
(Intercept)       66.06923 15.578716 1257   4.24099  0.0000
agedays            5.14534  0.227726 1257  22.59448  0.0000
knots...knot.100  11.96904  0.346943 1257  34.49860  0.0000
knots...knot.200  -2.57741  0.315231 1257  -8.17628  0.0000
knots...knot.300 -11.37429  0.386183 1257 -29.45308  0.0000
knots...knot.400  -2.11200  0.535636 1257  -3.94298  0.0001
knots...knot.500   0.48214  0.658743 1257   0.73190  0.4644
knots...knot.600   0.42001  0.706652 1257   0.59437  0.5524
...
```



```
> m.l2 <- lme(tail ~ agedays + knots((1:4) * 100, agedays), random = ~agedays |
+     animal, data = wal)
> ml.l1 <- update(m.l1, method = "ML")
> ml.l2 <- update(m.l2, method = "ML")
> anova(ml.l1, ml.l2)

      Model df      AIC      BIC    logLik   Test  L.Ratio p-value
ml.l1     1 12 17237.59 17299.99 -8606.795
ml.l2     2 10 17237.35 17289.35 -8608.673 1 vs 2 3.755532  0.1529

> m.l3 <- lme(tail ~ agedays + knots(c(50, 100, 150, 200, 300,
+     400), agedays), random = ~agedays | animal, data = wal)
> summary(m.l3)

...
                     Value Std.Error   DF    t-value p-value
(Intercept)       18.170507 19.205429 1257   0.946113  0.3443
agedays            6.402426  0.501652 1257  12.762672  0.0000
knots...knot.50    1.055724  0.788990 1257   1.338070  0.1811
knots...knot.100   1.895643  0.709919 1257   2.670224  0.0077
knots...knot.150  15.056135  0.680427 1257  22.127467  0.0000
knots...knot.200 -11.422528  0.470835 1257 -24.260131  0.0000
knots...knot.300  -9.635222  0.306205 1257 -31.466547  0.0000
knots...knot.400  -2.005982  0.242039 1257  -8.287855  0.0000
...

> ml.l3 <- update(m.l3, method = "ML")
> anova(ml.l2, ml.l3)

      Model df      AIC      BIC    logLik   Test  L.Ratio p-value
ml.l2     1 10 17237.35 17289.35 -8608.673
ml.l3     2 12 16802.08 16864.48 -8389.037 1 vs 2 439.2709  <.0001
```


5000
4000
3000
2000
1000
0
tail
0
200
400
600
agedays

Fitted/Predicted Values

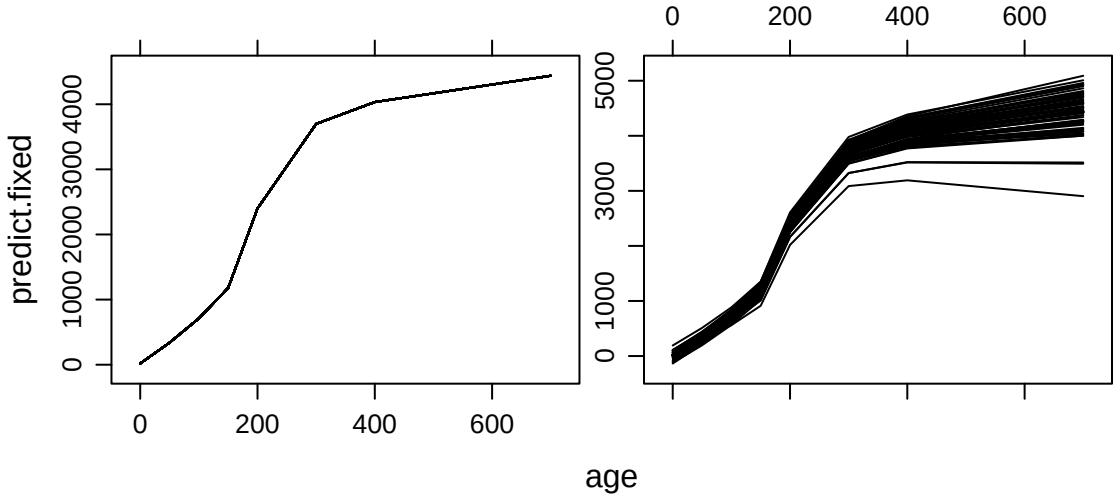
`fitted` gives the fitted (or predicted) values for the values in the data set

```
> fit <- fitted(m.l3, level = 0:1)
> fit <- data.frame(id = wal$animal, age = wal$agedays, fit)
> head(fit)

  id age    fixed    animal
1 45  14 107.8045  43.58001
2 45  28 197.4384 135.36642
3 45  49 331.8894 273.04605
4 45  69 479.9966 424.22825
5 45  83 584.4107 530.79481
6 45  97 688.8248 637.36136
```

`predict` will give the fitted values for values not in the data set

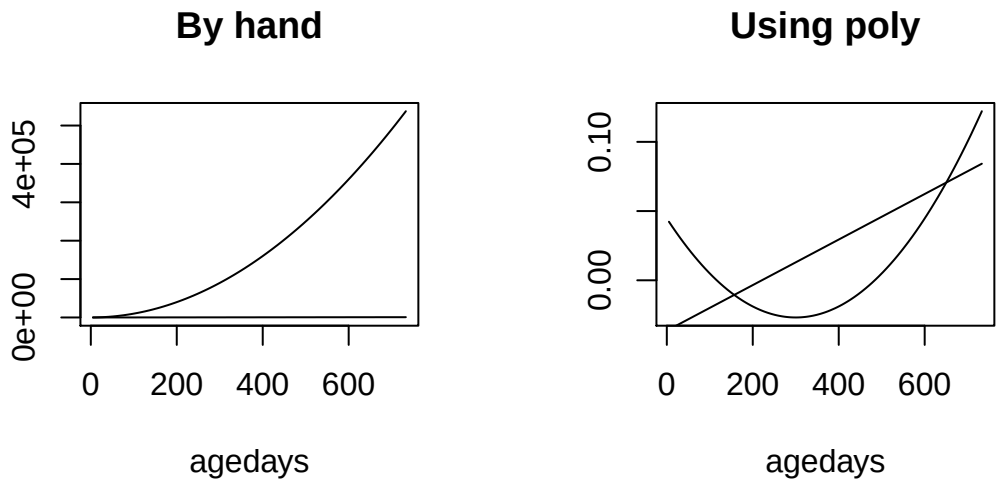
```
> newd <- expand.grid(agedays = c(0, 50, 100, 150, 200, 300, 400,
+     500, 600, 700), animal = c(44, 45))
> data.frame(agedays = newd$agedays, predict(m.l3, newdata = newd,
+     level = 0:1))

   agedays animal predict.fixed predict.animal
1        0     44      18.17051       34.37618
2       50     44     338.29180      328.34709
3      100     44     711.19929      675.10420
4      150     44    1178.88892     1116.64345
5      200     44    2399.38530     2310.98944
6      300     44    3698.12521     3557.42859
7      400     44    4033.34295     3840.34556
8      500     44    4167.96247     3922.66433
9      600     44    4302.58200     4004.98309
10     700     44    4437.20153     4087.30185
11       0     45      18.17051      -48.20640
12      50     45     338.29180      279.60222
13     100     45     711.19929      660.19705
14     150     45    1178.88892     1135.57401
15     200     45    2399.38530     2363.75772
16     300     45    3698.12521     3677.87230
17     400     45    4033.34295     4028.46471
18     500     45    4167.96247     4178.45890
19     600     45    4302.58200     4328.45309
20     700     45    4437.20153     4478.44729
```

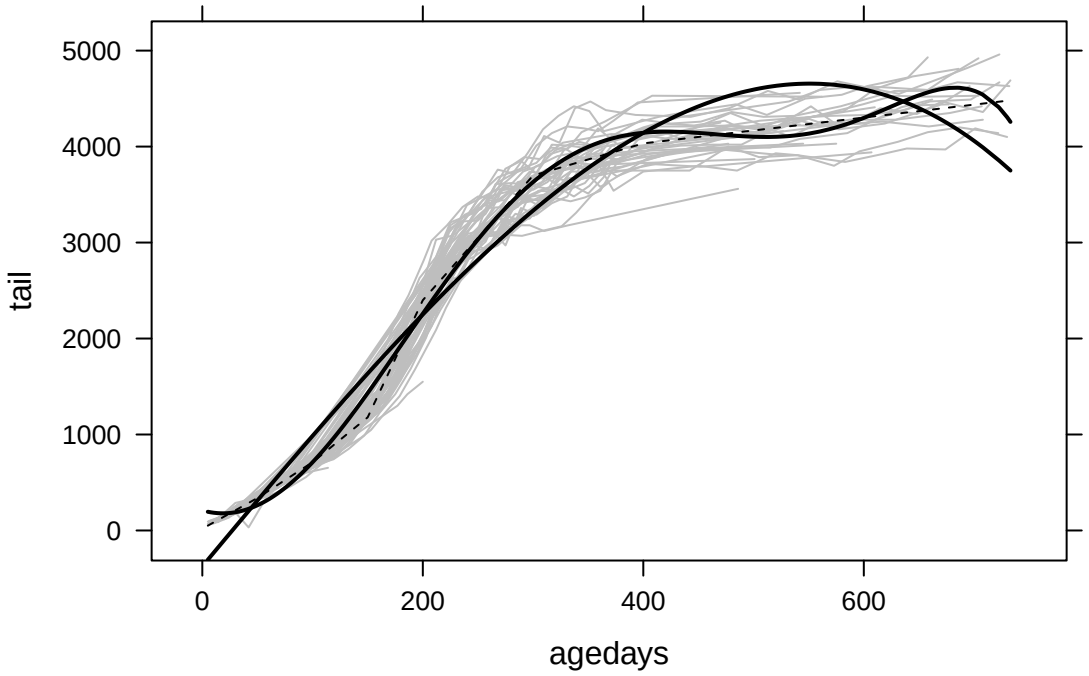
```
> newd <- expand.grid(agedays = c(0, 50, 100, 150, 200, 300, 400,
+     500, 600, 700), animal = levels(wal$animal))
> pred <- data.frame(age = newd$agedays, predict(m.l3, newd, level = 0:1))
> p1 <- xyplot(predict.fixed ~ age, group = animal, type = "l",
+     par.settings = black, data = pred)
> p2 <- xyplot(predict.animal ~ age, group = animal, type = "l",
+     par.settings = black, data = pred)
> plot(c(p1, p2))
```



Polynomial

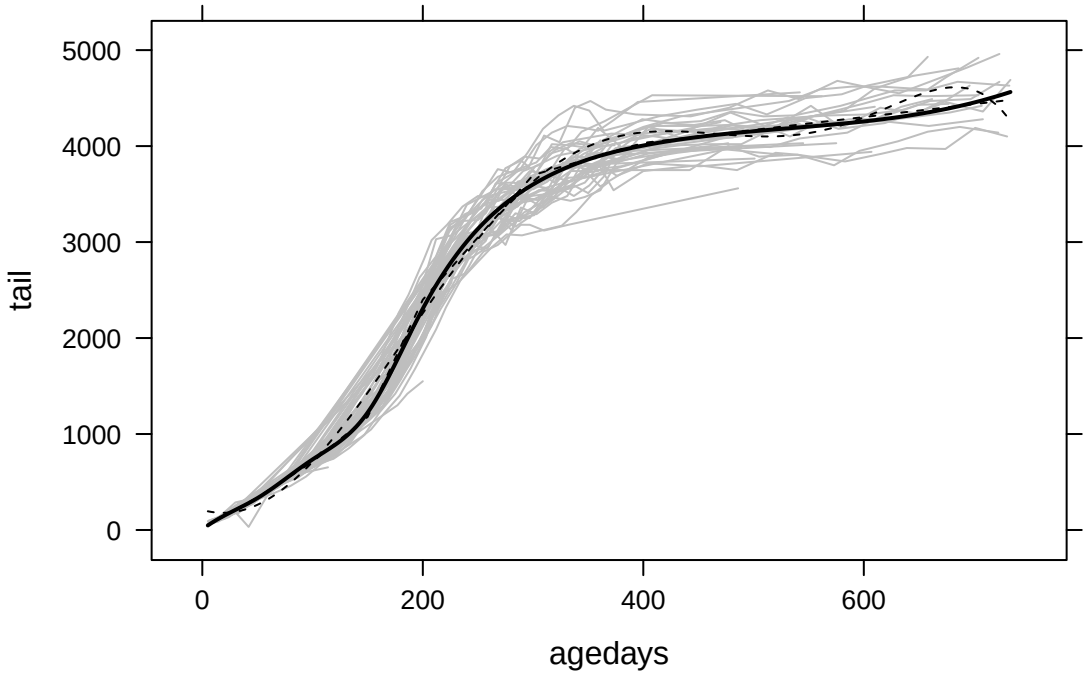
```
> m.p2a <- lme(tail ~ agedays + I(agedays^2), random = ~agedays |
+     animal, data = wal, method = "ML")
> summary(m.p2a)

...
                 Value Std.Error   DF   t-value p-value
(Intercept)  -539.1802 21.461658 1262 -25.12295       0
agedays        17.2507  0.179214 1262  96.25781       0
I(agedays^2)   -0.0147  0.000271 1262 -54.32359       0
...

> poly2 <- poly(wal$agedays, 2)
> head(poly2)

               1           2
[1,] -0.03407978 0.038154803
[2,] -0.03177639 0.031946559
[3,] -0.02832130 0.023218274
[4,] -0.02503074 0.015557249
[5,] -0.02272734 0.010572793
[6,] -0.02042395 0.005899847
```




5000
4000
3000
2000
1000
0
tail
0
200
400
600
agedays

Cubic Spline

```
> m.s1 <- lme(tail ~ poly(agedays, 3) + knots(c(50, 100, 150, 200,
+     300, 400), agedays, 3), random = ~agedays | animal, data = wal)
> summary(m.s1)

...
                       Value Std.Error   DF    t-value p-value
(Intercept)          27814.1   21387.6 1255   1.300476  0.1937
poly(agedays, 3)1  1790286.7 1445445.2 1255   1.238571  0.2157
poly(agedays, 3)2  1090960.2  921948.2 1255   1.183320  0.2369
poly(agedays, 3)3   212536.5  186719.2 1255   1.138268  0.2552
knots...knot.50          0.0       0.0 1255  -1.537660  0.1244
knots...knot.100         0.0       0.0 1255   6.323144  0.0000
knots...knot.150         0.0       0.0 1255 -16.275667  0.0000
knots...knot.200         0.0       0.0 1255  21.178691  0.0000
knots...knot.300         0.0       0.0 1255  -4.210058  0.0000
knots...knot.400         0.0       0.0 1255  -3.643160  0.0003
...
```



B-spline

```
> library(splines)
> m.bs10 <- lme(tail ~ bs(agedays, 10), random = ~agedays | animal,
+     data = wal)
> summary(m.bs10)

...
                    Value Std.Error   DF  t-value p-value
(Intercept)         54.410  30.34208 1254  1.79321  0.0732
bs(agedays, 10)1    82.833  50.49721 1254  1.64035  0.1012
bs(agedays, 10)2   212.912  30.07494 1254  7.07939  0.0000
bs(agedays, 10)3   612.171  39.92065 1254 15.33469  0.0000
bs(agedays, 10)4   887.714  34.78825 1254 25.51764  0.0000
bs(agedays, 10)5  2109.383  38.03162 1254 55.46392  0.0000
bs(agedays, 10)6  3270.694  38.19814 1254 85.62442  0.0000
bs(agedays, 10)7  3685.231  42.28260 1254 87.15715  0.0000
bs(agedays, 10)8  4316.317  61.54901 1254 70.12813  0.0000
bs(agedays, 10)9  4023.968  75.16492 1254 53.53518  0.0000
bs(agedays, 10)10 4539.785  77.16347 1254 58.83335  0.0000
...
```